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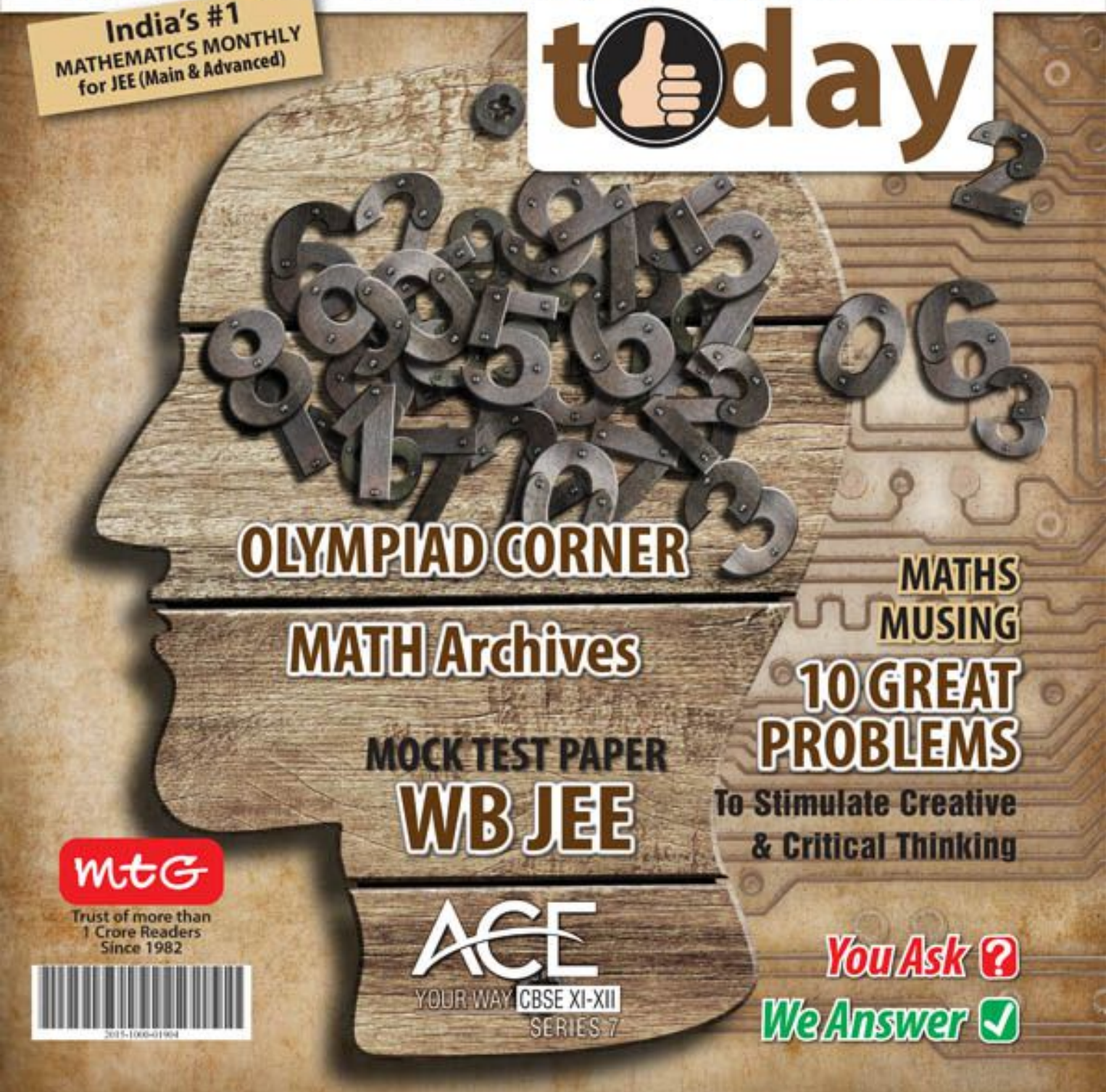

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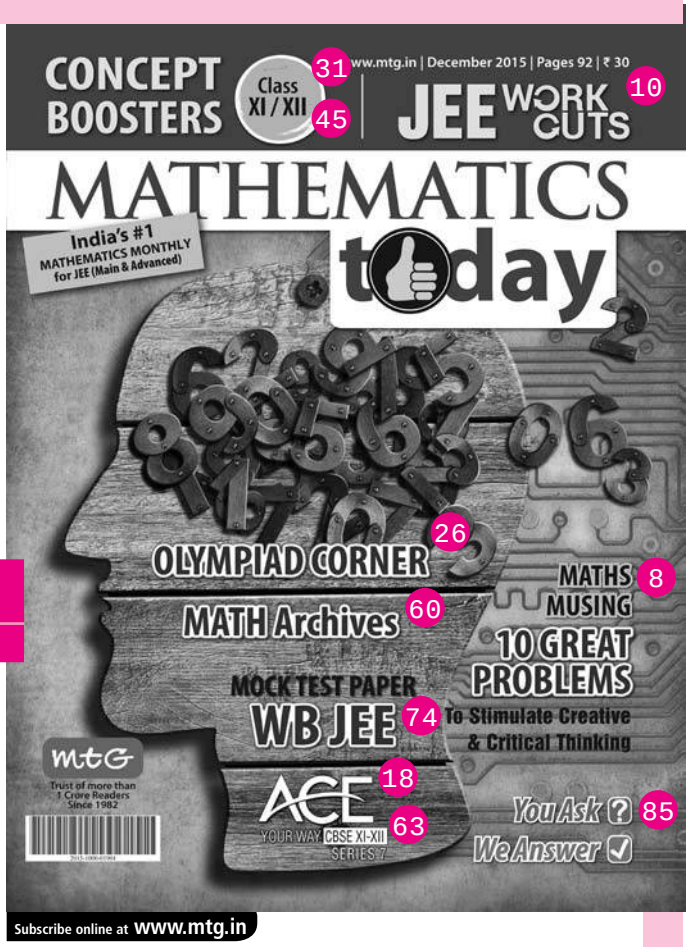


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# MATHS MUSING

*Maths Musing was started in January 2003 issue of Mathematics Today with the suggestion of Shri Mahabir Singh. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.*
*During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.*
*Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.*

**Prof. Dr. Ramanaiah Gundala,** Former Dean of Science and Humanities, Anna University, Chennai

## PROBLEM Set 156

### JEE MAIN

**1.** Let $0 < a < b$ be integers such that the H.M. of $a$ and $b$ is 2016. The number of pairs $(a, b)$ is
(a) 64 (b) 65 (c) 66 (d) 67

**2.** Let $0 < a < b$ be integers such that $\frac{1}{a}+\frac{1}{b}=\frac{1}{2016}$. The number of pairs $(a, b)$ is
(a) 80 (b) 81 (c) 82 (d) 85

**3.** The sum of all the roots of the equation $\left(1-\frac{2}{5x}\right)^{2016}=1$ is
(a) 402 (b) 403 (c) 404 (d) 405

**4.** Let $(10^{2016}+5)^2 = 225\,N$. If $S$ is the sum of the digits of $N$, then the sum of the digits of $S$ is
(a) 16 (b) 17 (c) 18 (d) 20

**5.** $\int_0^{2016} x^5(2016-x)^6\,dx =$
(a) $\frac{2}{11}(2016)^{11}$ (b) $\frac{4}{11}(2016)^{11}$
(c) $\frac{2}{11}(2016)^{12}$ (d) $\frac{4}{11}(2016)^{12}$

### JEE ADVANCED

**6.** Let $N$ be the number of right angled triangles with integer sides and inradius $r = 2016$. Then $N$ is divisible by
(a) 2 (b) 3 (c) 5 (d) 7

### COMPREHENSION

Two numbers $a$ and $b$ are selected from the set of the first 25 natural numbers. The number of ways of selecting them such that

**7.** $a^2 - b^2$ is divisible by 5 is
(a) 75 (b) 100 (c) 115 (d) 150

**8.** $a^4 - b^4$ is divisible by 5 is
(a) 150 (b) 200 (c) 250 (d) 300

### INTEGER MATCH

**9.** Twenty five identical marbles are to be distributed among 3 children so that each gets atleast 4. The probability that no two get equal number is $\frac{m}{n}$, reduced fraction, where $(n - m)$ is

### MATCHING LIST

**10.**

| | Column-I | | Column-II |
|---|---|---|---|
| P. | $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=1$. The maximum value of $\lvert4\vec{a}-5\vec{b}\rvert^2+\lvert4\vec{b}-5\vec{c}\rvert^2+\lvert4\vec{c}-5\vec{a}\rvert^2$ is | 1. | 173 |
| Q. | If $x^2 + x + 1 = 0$, then $\sum_{r=1}^{100}\left(x^r+\frac{1}{x^r}\right)^3$ is | 2. | 180 |
| R. | If $y = (x^2 - x + 1)^5(x - 1)^4(x - 2)^3 (x - 3)^3$, then $\frac{d^4y}{dx^4}$ at $x = 1$ is | 3. | 183 |
| S. | If $\alpha, \beta$ are the roots of $x^2 - 3x + 5 = 0$, $r, s$ are the roots of $x^2 + 5x - 3 = 0$ and $\alpha r + \beta s$, $\alpha s + \beta r$ are the roots of $x^2 + px + q = 0$, then $(p + q)$ is | 4. | 192 |
| | | 5. | 197 |

| | P | Q | R | S |
|---|---|---|---|---|
| (a) | 3 | 5 | 4 | 1 |
| (b) | 1 | 2 | 3 | 4 |
| (c) | 2 | 3 | 4 | 5 |
| (d) | 1 | 3 | 4 | 5 |

See Solution set of Maths Musing 155 on page no. 84

**Prof. Ramanaiah is the author of MTG JEE(Main & Advanced) Mathematics series**

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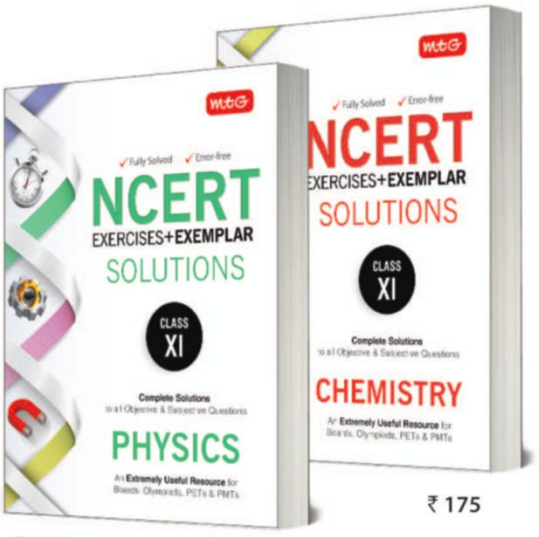


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# JEE WORK OUTS

## PAPER-1

### SECTION-1

**This section contains 8 questions. The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.**

**1.** If the equation of the line(s) through the point of intersection of the lines $x - 3y + 1 = 0$, $2x + 5y - 9 = 0$ and whose distance from the origin is $\sqrt{5}$ is $2x + y = k$, then determine $k$.

**2.** If two perpendicular tangents $PA$ and $PB$ are drawn to $y^2 = 7x$ then find the minimum length of chord $AB$.

**3.** The line $x + 2y + a = 0$ intersects the circle $x^2 + y^2 - 4 = 0$ at two distinct points $A$ and $B$. Another line $12x - 6y - 41 = 0$ intersect the circle $x^2 + y^2 - 4x - 2y + 1 = 0$ at two distinct points $C$ and $D$. The value of '$a$' for which the four points $A$, $B$, $C$ and $D$ are concyclic is

**4.** The area bounded by the curves $x + 2|y| = 1$ and $x = 0$ is $A$, then $4A =$

**5.** If from point $P(4, 4)$ perpendiculars to the straight lines $3x + 4y + 5 = 0$ and $y = mx + 7$ meet at $Q$ and $R$ and area of triangle $PQR$ is maximum. Then $3m$ is equal to

**6.** If $p$ is the length of the perpendicular from a focus upon the tangent at any point $P$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $r$ is the distance of $P$ from the focus, then $\frac{2a}{r}-\frac{b^2}{p^2}=$

**7.** The number of different hyperbolas represented by the equation $(5x - 5)^2 + (5y + 5)^2 = {}^mC_n(3x + 4y - 2)^2$ where $m$ and $n$ are integers satisfying $1 \le n \le m \le 5$ is

**8.** Shortest distance between circle $(8x - 33)^2 + 64y^2 = 1$ and parabola $4y^2 = x$ is $\frac{\sqrt{65}-k}{8}$, where $k$ is

### SECTION-2

**This section contains TEN questions. Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option (s) is (are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened, 0 If none of the bubbles is darkened, −2 In all other cases.**

**9.** Let $e(l)$ be the eccentricity of $(x - 3)(y + 2) = l^2$, then the value of $e(2) - e(3)$ is

(a) 0 (b) 1 (c) 3 (d) −3

**10.** $P$ is a point which moves in the $xy$ plane such that the point $P$ is nearer to the centre of a square than any of the sides. The four vertices of the square are $(\pm a, \pm a)$. The region in which $P$ will move is bounded by parts of parabola of which one has the equation.

(a) $y^2 = a^2 + 2ax$ (b) $x^2 = a^2 + 2ay$

(c) $y^2 + 2ax = a^2$ (d) $x^2 = a^2 - 2ay$

**11.** The straight line segment are $P(1, 3)$ and $Q(1, 1)$. $R$ is a point on the line segment $PQ$ such that $PR : QR = 1 : \lambda$. If $R$ is an interior point of the parabola $y^2 = 4x$, then $\lambda$ can take value in the interval

(a) $(0, 1)$ (b) $\left(-\frac{3}{5}, 1\right)$

(c) $\left(\frac{1}{2}, \frac{3}{5}\right)$ (d) none of these

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**12.** If $PSQ$ and $PS'R$ be two focal chords of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $P$, $Q$, $R$ are points $(a\cos\theta, b\sin\theta)$, $(a\cos\theta_1, b\sin\theta_1)$, $(a\cos\theta_2, b\cos\theta_2)$ respectively, then

(a) $\tan\frac{\theta}{2}\tan\frac{\theta_1}{2}=\frac{e-1}{e+1}$ (b) $\tan\frac{\theta}{2}\tan\frac{\theta_1}{2}=\frac{e^2+1}{e^2-1}$

(c) $\tan\frac{\theta}{2}\tan\frac{\theta_2}{2}=\frac{1+e}{1-e}$ (d) $\tan\frac{\theta}{2}\tan\frac{\theta_2}{2}=\frac{1+e^2}{1-e^2}$

**13.** The locus of middle points of the normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is

(a) $(y^2 - x^2)^3 = 4a^2x^2y^2$ (b) $(y^2 - x^2)^2 = 4a^2x^2y^2$

(c) $(y^2 + x^2)^3 = 4a^2x^2y^2$ (d) $(y^2 + x^2)^2 = 4a^2x^2y^2$

**14.** The equation of the tangent to the hyperbola $\frac{x^2}{4}-\frac{y^2}{3}=1$, parallel to the line $y = x + 2$ is

(a) $y = -x + 1$ (b) $y = x + 1$

(c) $y = -x - 1$ (d) $y = x - 1$

**15.** Eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance of 2 units from the centre of the ellipse is

(a) $\pi/4$ (b) $\pi/3$ (c) $3\pi/4$ (d) $2\pi/3$

**16.** On the ellipse $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are

(a) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$

(c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (d) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

**17.** Two circles $x^2 + y^2 = 6$ and $x^2 + y^2 - 6x + 8 = 0$ are given. Then the equation of the circle through their points of intersection and the point $(1, 1)$ is

(a) $x^2 + y^2 - 6x + 4 = 0$ (b) $x^2 + y^2 - 3x + 1 = 0$

(c) $x^2 + y^2 - 4y + 2 = 0$ (d) none of these

**18.** If $p_1$, $p_2$ denote the lengths of the perpendicular from the point $(2, 3)$ on the lines given by $15x^2 + 31xy + 14y^2 = 0$, then

(a) $p_1 + p_2 = \frac{31}{14}$ (b) $|p_1 - p_2| = \frac{31}{\sqrt{74}} - \frac{12}{\sqrt{13}}$

(c) $p_1p_2 = \frac{372}{\sqrt{962}}$ (d) $p_1p_2 = \frac{15}{14}$

## SECTION-3

**This section contains TWO questions. Each question contains two columns, Column I and Column II. Column I has four entries (A), (B), (C) and (D), Column II has four entries (P), (Q), (R) and (S). Match the entries in Column I with the entries in Column II. One or more entries in Column I may match with one or more entries in Column II. The ORS contains a 4 × 4 matrix whose layout will be similar to the one shown below.**

(A) P Q R S T

(B) P Q R S T

(C) P Q R S T

(D) P Q R S T

**For each entry in Column I, darken the bubbles of all the matching entries. For example, if entry (A) in Column I matches with entries (Q) and (R) then darken these two bubbles in the ORS. Similarly, for entries (B), (C) and (D). Marking scheme : For each entry in Column I. +2 If only the bubble(s) corresponding to all the correct match(es) is (are) darkened, 0 If none of the bubbles is darkened, –1 In all other cases.**

**19.** Match the following:

| | Column I | | Column II |
|---|---|---|---|
| (A) | The circles $x^2 + y^2 + 2x + c = 0$ and $x^2 + y^2 + 2y + c = 0$ touch each other if | P. | $c = 1$ |
| (B) | The circles $x^2 + y^2 + 2x + 3y + c = 0$ and $x^2 + y^2 - x + 2y + c = 0$ intersect orthogonally if | Q. | $c < 2$ |
| (C) | The circle $x^2 + y^2 = 9$ contains the circle $x^2 + y^2 - 2x + 1 - c^2 = 0$ for | R. | $c = 1/2$ |
| (D) | The circle $x^2 + y^2 = 9$ is contained in the circle $x^2 + y^2 - 6x - 8y + 25 - c^2 = 0$ if | S. | $c > 8$ |

**20.** Match the following:

| | Column I | | Column II |
|---|---|---|---|
| (A) | The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at $(0, 3)$ is | P. | $\frac{2}{\sqrt{3}}$ |
| (B) | If the length of the major axis of an ellipse is three times the length of its minor axis, then its eccentricity is | Q. | $\frac{2\sqrt{2}}{3}$ |
| (C) | The eccentricity of hyperbola whose latus rectum is 8 and conjugate axis is half the distance between foci, is | R. | $\sqrt{\frac{17}{20}}$ |
| (D) | The value of $m$ for which $y = mx + 6$ is a tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{49}=1$ is | S. | 4 |

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## SECTION-1

**This section contains EIGHT questions. The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive. For each question, darken the bubble corresponding to the correct integer in the ORS. Marking scheme : +4 If the bubble corresponding to the answer is darkened, 0 In all other cases.**

**1.** If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then value of $|m|$ is

**2.** Find the length of the latus rectum of the conic passing through the origin and having property that normal at each point $(x, y)$ intersect the $x$-axis at $((x + 1), 0)$.

**3.** If the circles which can be drawn to pass through (1, 0) and (3, 0) and touches the $y$-axis, intersect at an angle $\theta$ then $4\cos\theta$ is equal to

**4.** If $m$ is the slope of a common tangent of the parabola $y^2 = 16x$ and the circle $x^2 + y^2 = 8$, then $m^2$ is equal to

**5.** If both the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then find the value of $b^2$.

**6.** Let $P_i$ and $P_i'$ be the feet of the perpendicular drawn from foci $S$, $S'$ on a tangent $T_i$, for $i = 1, 2, ..., 10$ to an ellipse whose length of semi major axis is 20, if $\sum_{i=1}^{10}(SP_i)(S'P_i') = 2560$, then find the value of $5e$ (where '$e$' denotes eccentricity).

**7.** Two rays in the 1st quadrant $x + y = |a|$ and $ax - y = 1$ intersect each other in the interval $a \in (a_0, \infty)$ then the value of $a_0$ is

**8.** The distance from the centre of the circle $x^2 + y^2 = 2x$ to the straight line passing through the point of intersection of the circles $x^2 + y^2 + 5x - 8y + 1 = 0$ and $x^2 + y^2 - 3x + 7y - 25 = 0$ is

## SECTION-2

**This section contains EIGHT questions. Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened, 0 If none of the bubbles is darkened, –2 In all other cases.**

**9.** If the tangent at the point $(a\sec\alpha, b\tan\alpha)$ to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ meets the transverse axis at $T$, then the distance of $T$ from a focus of the hyperbola is

(a) $a(e - \cos\alpha)$ (b) $b(e + \cos\alpha)$

(c) $a(e + \cos\alpha)$ (d) $\sqrt{a^2e^2 + b^2\cot^2\alpha}$

**10.** If $y = mx$ be one of the bisectors of the lines $2x^2 - 3xy - 4y^2 = 0$ then

(a) $m=\frac{-30-\sqrt{41}}{8}$ (b) $m=\frac{-3+\sqrt{41}}{8}$

(c) $m = 2+\sqrt{5}$ (d) $m = 2-\sqrt{5}$

**11.** The coordinates of an end point of the latus rectum of the parabola $(y - 1)^2 = 2(x + 2)$ are

(a) (–2, 1) (b) (–3/2, 1)

(c) (–3/2, 2) (d) (–3/2, 0)

**12.** Consider the circle $C_1 : x^2 + y^2 = 4$ and the circle $C_2 : x^2 + y^2 - 6x - 8y - 24 = 0$. Then

(a) $C_1$ and $C_2$ touch each other

(b) $C_1$ completely lies within $C_2$ without a common point

(c) $C_1$ and $C_2$ touch externally

(d) the number of common tangents is one

**13.** The line $y = 3x$ bisects the angle between the lines $a^2x^2 + 2axy + y^2 = 0$ if $a$ is equal to

(a) –3 (b) 3 (c) –1/3 (d) 1/3

**14.** An isosceles triangle $ABC$ is inscribed in a circle $x^2 + y^2 = a^2$ with the vertex $A$ at $(a, 0)$ and the base angle $B$ and $C$ each equal to 75°, then coordinates of an end point of the base are

(a) $\left(\frac{\sqrt{3}a}{2}, \frac{a}{2}\right)$ (b) $\left(-\frac{\sqrt{3}a}{2}, \frac{a}{2}\right)$

(c) $\left(-\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$ (d) $\left(\frac{\sqrt{3}a}{2}, -\frac{a}{2}\right)$

**15.** The equation of the pair of straight lines, each of which makes an angle $\alpha$ with the line $y = x$, is

(a) $x^2 + 2xy\sec^2\alpha + y^2 = 0$

(b) $x^2 + 2xy\,\text{cosec}^2\alpha + y^2 = 0$

(c) $x^2 - 2xy\,\text{cosec}^2\alpha + y^2 = 0$

(d) $x^2 - 2xy\sec^2\alpha + y^2 = 0$

**16.** Let $L_1$ be a straight line through the origin and $L_2$ be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ and $L_1$ and $L_2$ are equal, then which of the following equation can represent $L_1$?

(a) $x + 7y = 0$ (b) $x - y = 0$

(c) $x - 7y = 0$ (d) $x + 2y = 0$

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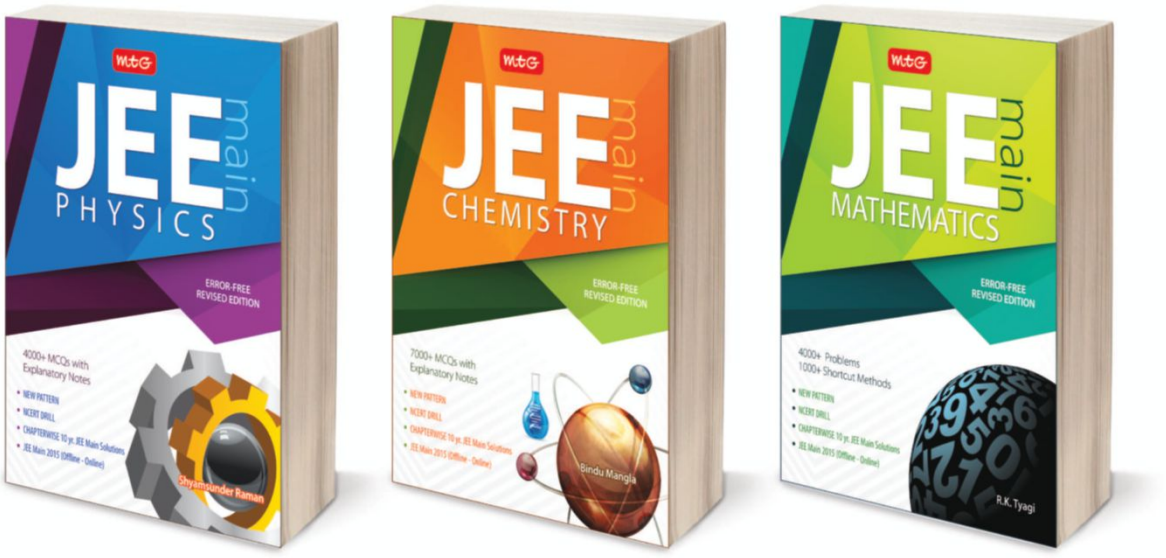


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## SECTION-3

**This section contains TWO paragraphs. Based on each paragraph, there will be TWO questions. Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is(are) correct. For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS. Marking scheme : +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened, 0 If none of the bubbles is darkened, –2 In all other cases**

### Paragraph for Q. No. 17 & 18

The equation of the normal to the parabola $y^2 = 4ax$ at the point $(x_1, y_1)$ is given by $y - y_1 = -\frac{y_1}{2a}(x - x_1)$.

Also the equation of normal in parametric form at $(at^2, 2at)$ is $y + tx = 2at + at^3$ and in the slope $(m)$ form it is $y = mx - 2am - am^3$. The point of intersection $(T)$ of normals at any two points on the parabola can be obtained by

$T \equiv ((2a + a(t_1^2 + t_2^2 + t_1t_2), -at_1t_2(t_1 + t_2)])$

**17.** The normal to the parabola $y^2 = 8x$ at the point (2, 4) meets it again at $(l, m)$. The length of the normal chord is

(a) 8 (b) $16\sqrt{2}$

(c) $8\sqrt{3}$ (d) $4\sqrt{5}$

**18.** If the normal to the parabola $y^2 = 4ax$, makes an angle θ with the axis, then it will cut the curve again at an angle

(a) $\sin^{-1}\left(\frac{1}{2}\sin\theta\right)$ (b) $\tan^{-1}(2\theta)$

(c) $\tan^{-1}\left(\frac{1}{2}\tan\theta\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\sin 2\theta\right)$

### Paragraph for Q. No. 19 & 20

Suppose the Point $(a\cos\theta, b\sin\theta)$ satisfies the equation of the ellipse for any value of 'θ'. This 'θ' is also an eccentric angle. Equation of tangent at 'θ' is given by $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ and also the equation of tangent to the ellipse in terms of slope '$m$' is $y = mx \pm \sqrt{a^2m^2 + b^2}$

**19.** A tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the axes in $A$ and $B$ respectively and touches the ellipse at any point $P$ in the first quadrant. If the point $P$ divides $AB$ in the ratio 3 : 1, then equation of tangent is

(a) $ax + by = \sqrt{3}ab$ (b) $bx + ay = ab$

(c) $bx + a\sqrt{3}y = 2ab$ (d) $b\sqrt{3}x + ay = 3ab$

**20.** The equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$ is

(a) $x^2 + y^2 = 22$ (b) $(x + 1)^2 + (y + 1)^2 = 3$

(c) $(x-1)^2 + (y-1)^2 = \frac{1}{3}$

(d) $(x-1)^2 + y^2 = \frac{11}{3}$

## ANSWER KEYS

### PAPER-1

1. (5) 2. (7) 3. (2) 4. (2) 5. (4)
6. (1) 7. (6) 8. (8) 9. (a)
10. (a, b, c, d) 11. (a, b, c)
12. (a, c) 13. (a) 14. (b, d) 15. (a, c) 16. (b, d)
17. (b) 18. (b, c) 19. (A) → (r); (B) → (p); (C) → (p, q, r); (D) → (s)
20. (A) → (s); (B) → (q); (C) → (p); (D) → (r)

### PAPER-2

1. (1) 2. (2) 3. (2) 4. (1) 5. (7)
6. (3) 7. (1) 8. (2) 9. (a, c) 10. (c, d)
11. (c, d) 12. (a, d) 13. (a) 14. (b, c) 15. (d)
16. (a, b) 17. (b) 18. (c) 19. (c) 20. (d)

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## When Hollywood meets the math genius from India

A movie on a Indian mathematician Ramanujan is going to hit the big screen. Matthew Brown's biopic-- The Man Who Knew Infinity - is set for premier on September 17 at the Toronto Film Festival.

A week later, it will be the opening film at the Zurich Film Festival. The film stars Jeremy Irons, Dev Patel, Toby Jones, Stephen Fry, Devika Bhise, Jeremy Northam and Kevin McNally.

The nearly one hour 40 minutes film tells the true story of Srinivasa Ramanujan (Patel), a self-taught mathematics genius from India, who leaves behind his young bride, Janaki (Bhise), to travel across the world to Cambridge, England. Brown's screenplay was adapted from Robert Kanigel's biography "The Man Who Knew Infinity: A Life of the Genius Ramanujan". It is not a typical Hollywood Film and connects deeply with the people. The film will help us know more about Ramanujan," film director Matt Brown told HT. "I came across the book 10 years ago and was very inspired by it. I instantly wanted to make a movie. It has taken me 10 years to make the film. But I am so happy that I could finally do it," he said. Matt said that he had not brought any changes in the film. "I have made sure that the human factor was at the forefront. Math is difficult to understand and one has to connect at a different emotional level."

*Courtesy : Hindustan Times*

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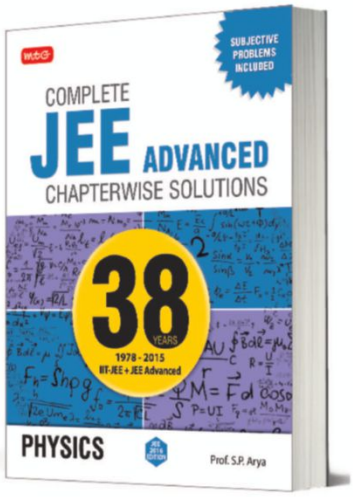


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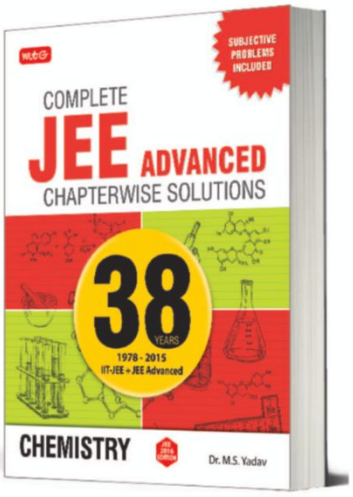


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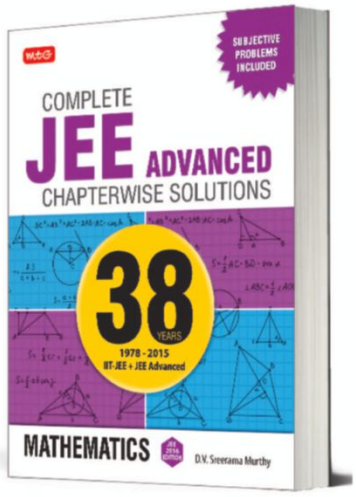


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# ACE YOUR WAY CBSE XI

## LIMITS AND DERIVATIVES & MATHEMATICAL REASONING

## LIMITS AND DERIVATIVES

### LIMITS

#### Left hand limit

The left hand limit of $f$ at $x = a$ is the expected value of $f$ at $x = a$ given the values of $f$ near $x$ to the left of $a$. It is denoted by $\lim_{x \to a^-} f(x)$.

#### Right hand limit

The right hand limit of $f$ at $x = a$ is the expected value of $f$ at $x = a$ given the values of $f$ near $x$ to the right of $a$. It is denoted by $\lim_{x \to a^+} f(x)$.

#### Limit of a function at a Point

Limit of a function at $x = a$ is the value of $f(x)$ at $x = a$, *i.e.*, $\lim_{x \to a} f(x)$.

***Note :***

$\lim_{x \to a} f(x)$ exists at $x = a$ iff $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$

### ALGEBRA OF LIMITS

Let $f$ and $g$ be two functions such that both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

- $\lim_{x \to a}\{f(x) + g(x)\} = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
- $\lim_{x \to a}\{f(x) - g(x)\} = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
- $\lim_{x \to a}\{c \cdot f(x)\} = c \cdot \lim_{x \to a} f(x)$, where $c$ is a constant.
- $\lim_{x \to a}\{f(x) \cdot g(x)\} = \left\{\lim_{x \to a} f(x)\right\} \cdot \left\{\lim_{x \to a} g(x)\right\}$
- $\lim_{x \to a}\left\{\frac{f(x)}{g(x)}\right\} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, provided $\lim_{x \to a} g(x) \neq 0, g(x) \neq 0$

### LIMITS OF POLYNOMIALS AND RATIONAL FUNCTIONS

- A function $f(x)$ is said to be a polynomial function if $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$, $0 \leq i \leq n$ where $a_i$'s are real numbers.

  If $a_i = 0 \; \forall \; 0 \leq i \leq n$, then $f(x)$ is called zero function.

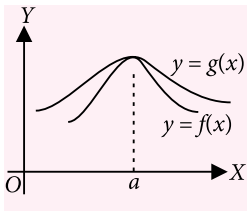
  Also, $\lim_{x \to a} f(x) = f(a)$

- A function $f(x)$ is said to be a rational function, if $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials in $x$ and $h(x) \neq 0$

  Then $\lim_{x \to a} f(x) = \frac{\lim_{x \to a} g(x)}{\lim_{x \to a} h(x)} = \frac{g(a)}{h(a)}$

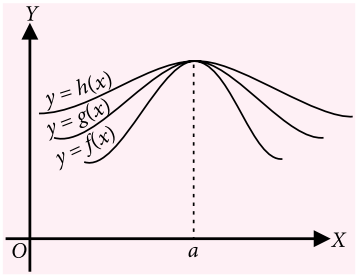
### SOME IMPORTANT THEOREMS

**1.** If $f$ and $g$ be two real valued functions such that $f(x) \leq g(x) \; \forall \; x$ lies in the common domain of $f$ and $g$, then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$,




when both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist at point '$a$'.

**2.** **Sandwich Theorem :** If $f(x)$, $g(x)$ and $h(x)$ are real functions of $x$ such that $f(x) \leq g(x) \leq h(x)$ for all $x$ lies in the common domain of $f$, $g$ and $h$ and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = l$ (say), for some real number $a$, then $\lim_{x \to a} g(x) = l$.



## SOME STANDARD LIMITS

**(i)** $\lim\limits_{x\to a}\dfrac{x^n-a^n}{x-a}=na^{n-1}$, for any positive integer $n$.

**(ii)** $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

**(iii)** $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}=0$

**(iv)** $\lim\limits_{x\to a}\dfrac{\sin(x-a)}{x-a}=1$

**(v)** $\lim\limits_{x\to 0}\dfrac{\tan x}{x}=1$

**(vi)** $\lim\limits_{x\to a}\dfrac{\tan(x-a)}{x-a}=1$

## SOME ADDITIONAL RESULTS BASED ON LIMITS OF EXPONENTIAL AND LOGARITHMIC FUNCTIONS

**(i)** $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}=1$

**(ii)** $\lim\limits_{x\to 0}\dfrac{\log_e(1+x)}{x}=1$

**(iii)** $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\log_e a,\ a>1$

**(iv)** $\lim\limits_{x\to 0}(1+x)^{1/x}=e$

## DERIVATIVES

### Derivative of a function at a point

Let $f(x)$ be a real valued function and $a$ be any point in its domain of definition. The derivative of $f(x)$ at $x=a$ is denoted by $f'(a)$ and defined as $f'(a)=\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$, provided this limit exists.

## FIRST PRINCIPLE OF DERIVATIVE

Let $f(x)$ be a function finitely differentiable at every point on its domain. Then, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ is called the first principle of derivative. This method is also called ab-initio method or delta method.

## ALGEBRA OF DERIVATIVE OF FUNCTIONS

Let $f$ and $g$ be two functions such that their derivatives are defined in a common domain. Then

**(i)** $\dfrac{d}{dx}[f(x)+g(x)]=\dfrac{d}{dx}f(x)+\dfrac{d}{dx}g(x)$

**(ii)** $\dfrac{d}{dx}[f(x)-g(x)]=\dfrac{d}{dx}f(x)-\dfrac{d}{dx}g(x)$

**(iii)** $\dfrac{d}{dx}[f(x)\cdot g(x)]=g(x)\cdot\dfrac{d}{dx}f(x)+f(x)\cdot\dfrac{d}{dx}g(x)$ (product rule)

**(iv)** $\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right)=\dfrac{g(x)\dfrac{d}{dx}f(x)-f(x)\dfrac{d}{dx}g(x)}{(g(x))^2}$ (quotient rule)

## DERIVATIVE OF A POLYNOMIAL FUNCTION

Let $f(x)=a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-2}+\ldots.+a_1x+a_0$ be a polynomial function of degree $n$ in $x$ where $a_i\in R$, $i=0,1,2,\ldots.,n$ and $a_n\neq 0$. Then

$$\frac{d}{dx}f(x)=na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+(n-2)a_{n-2}x^{n-3}+\ldots\ldots\ldots+2a_2x+a_1$$

## DERIVATIVES OF SOME STANDARD FUNCTIONS

**(i)** $\dfrac{d}{dx}(x^n)=nx^{n-1}$

**(ii)** $\dfrac{d}{dx}(\sin x)=\cos x$

**(iii)** $\dfrac{d}{dx}(\cos x)=-\sin x$

**(iv)** $\dfrac{d}{dx}(\sec x)=\sec x\tan x,\left(x\neq n\pi+\dfrac{\pi}{2}\right)$

**(v)** $\dfrac{d}{dx}(\operatorname{cosec}x)=-\operatorname{cosec}x\cot x,(x\neq n\pi)$

**(vi)** $\dfrac{d}{dx}(c)=0$, where $c$ is independent of $x$.

**(vii)** $\dfrac{d}{dx}(\tan x)=\sec^2x,\left(x\neq n\pi+\dfrac{\pi}{2}\right)$

**(viii)** $\dfrac{d}{dx}(\cot x)=-\operatorname{cosec}^2x,(x\neq n\pi)$

# MATHEMATICAL REASONING

## STATEMENT

A sentence which is either true or false but not both is called a statement.

## NEGATION OF A STATEMENT

Let $p$ be any statement then the statement expressing denial of $p$ is called negation of $p$, which is denoted by $\sim p$ and read as 'not $p$'.

## SIMPLE AND COMPOUND STATEMENTS

**Simple statement :** A statement which has no other statement as its component, is called a simple statement.

**Compound statement :** It is a combination of two or more simple statements by using the words "and", "or" etc. In this case such statements are called component statement.

**Rule 1 :** The compound statement with "AND" is

**(a)** true, if all its component statements are true.

**(b)** false, if any or all of its component statements are false.

**Rule 2 :** The compound statement with an "OR" is

**(a)** true, when one component statement is true or both the component statements are true.

**(b)** false, when both component statements are false.

## QUANTIFIERS

Many mathematical statements contain phrases 'there exists' and 'for all' or 'for every'. These phrases are called quantifiers.

## IMPLICATIONS

In Mathematics, we come across many statements of the form "if $p$ then $q$", "$p$ only if $q$" and "if and only if", such statements are called implications.

- **If then implication :** A sentence "if $p$ then $q$" can be written in the following ways :
  - **(i)** $p$ implies $q$ (denoted by $p \Rightarrow q$)
  - **(ii)** $p$ is sufficient condition for $q$
  - **(iii)** $q$ is necessary condition for $p$
  - **(iv)** $p$ only if $q$
  - **(v)** $\sim q$ implies $\sim p$

- **If and only if Implication :**

If $p$ and $q$ are two statements, then the compound statement $p \Rightarrow q$ and $q \Rightarrow p$ is called if and only if implication and is denoted by $p \Leftrightarrow q$.

## CONVERSE STATEMENT

If $p$ and $q$ are two statements, then the converse of the implication "if $p$, then $q$" is "if $q$, then $p$".

## CONTRAPOSITIVE STATEMENT

If $p$ and $q$ are two statements, then the contrapositive of the implication "if $p$, then $q$" is "if not $q$, then not $p$" *i.e.*, "if $\sim q$ then $\sim p$".

## INVERSE

If $p$ and $q$ are two statements, then the inverse of "if $p$ then $q$" is "if $\sim p$ then $\sim q$."

## Very Short Answer Type

**1.** Evaluate $\lim\limits_{x \to 1/2} \dfrac{4x^2 - 1}{2x - 1}$

**2.** Evaluate $\lim\limits_{x \to 0} \dfrac{1 - \cos 2x}{x^2}$

**3.** Find the derivative of $\left(9x^2 + \dfrac{3}{x} + 5\sin x\right)$ w.r.t. $x$.

**4.** Evaluate $\lim\limits_{\theta \to 0} \theta \operatorname{cosec} \theta$

**5.** Which of the following sentences are statements? Give reasons for your answer.

(i) The square of a number is an even number.

(ii) All real numbers are complex numbers.

**6.** Find the component statement of the following compound statements and check whether they are true or false.

(i) The number 3 is prime or it is odd.

(ii) All integers are positive or negative.

## Short Answer Type

**7.** (i) $\lim\limits_{x \to a} \dfrac{x^5 - a^5}{x - a} = 405$, find all possible values of $a$.

(ii) Find the derivative of $e^x$ using first principle.

**8.** (i) Given below are two statements:

$p$ : 25 is a multiple of 5.

$q$ : 25 is a multiple of 8.

Write the compound statement, connecting these two statements with 'and' and 'or'. In both cases, check the validity of the compound statement.

(ii) If $f(x) = mx + c$ and $f(0) = f'(0) = 1$, find the value of $f(2)$.

**9.** Evaluate $\lim\limits_{x \to 0} \dfrac{\cos ax - \cos bx}{x^2}$

**10.** (i) Differentiate $\dfrac{\sin(x - a)}{\cos x}$ with respect to $x$.

(ii) Differentiate $x^2 \tan x$ with respect to $x$.

**11.** Evaluate $\lim\limits_{x \to \frac{\pi}{6}} \dfrac{\sqrt{3}\sin x - \cos x}{\left(x - \dfrac{\pi}{6}\right)}$

**12.** Are the following pair of statements negation of each other?

(i) The number $x$ is not a rational number.
The number $x$ is not an irrational number.

(ii) The number $x$ is not a rational number.
The number $x$ is an irrational number.

(iii) Two lines are not parallel.
Two lines does not intersect at a point.

## Long Answer Type

**13.** (i) Verify by the method of contradiction that $\sqrt{3}$ is irrational.

(ii) If $f(x)=\begin{cases} 5x-4, & 0<x\le 1 \\ 4x^3-3x, & 1<x<2 \end{cases}$, show that $\lim\limits_{x\to 1} f(x)$ exists.

**14.** Evaluate $\lim\limits_{x\to 0}\dfrac{\tan x+4\tan 2x-3\tan 3x}{x^2\tan x}$

**15.** Differentiate $x^2\cos x$ from first principle.

**16.** (i) If $y=x\tan x$, prove that $x\sin^2 x\dfrac{dy}{dx}=y^2+y\sin^2 x$.

(ii) If $y=\dfrac{x}{x+5}$, prove that $x\dfrac{dy}{dx}=y(1-y)$.

**17.** Using the words "necessary and sufficient" rewrite the statement "The integer $m$ is odd if and only if $m^3$ is odd". Also check whether the statement is true.

**18.** (i) Evaluate $\lim\limits_{x\to 2}\left(\dfrac{x^8-16}{x^4-4}+\dfrac{x^2-9}{x-3}\right)$

(ii) Evaluate $\lim\limits_{x\to 2}\dfrac{(2x+4)^{1/3}-2}{x-2}$

## SOLUTIONS

**1.** $\lim\limits_{x\to 1/2}\dfrac{4x^2-1}{2x-1}=\lim\limits_{x\to 1/2}\dfrac{(2x+1)(2x-1)}{2x-1}$

$=\lim\limits_{x\to 1/2}2x+1=2\left(\dfrac{1}{2}\right)+1=2$

**2.** $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2 x}{x^2}=2\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^2=2(1)^2=2$

**3.** We have $\dfrac{d}{dx}(9x^2+\dfrac{3}{x}+5\sin x)$

$=9\cdot\dfrac{d}{dx}(x^2)+3\cdot\dfrac{d}{dx}(x^{-1})+5\cdot\dfrac{d}{dx}(\sin x)$

$=(9\times 2x)+3\cdot(-1)x^{-2}+5\cos x=18x-\dfrac{3}{x^2}+5\cos x.$

**4.** $\lim\limits_{\theta\to 0}\theta\,\mathrm{cosec}\,\theta=\lim\limits_{\theta\to 0}\dfrac{\theta}{\sin\theta}=\lim\limits_{\theta\to 0}\dfrac{1}{\left(\dfrac{\sin\theta}{\theta}\right)}=\dfrac{1}{1}=1$

**5.** (i) The given sentence is not a statement, since the square of a number may be even or odd, *e.g.* $2^2$ = 4(even), $3^2$ = 9(odd)

(ii) The given sentence is a statement, since it is true that all real numbers are complex numbers of the form $a + i\,0$.

**6.** (i) $p$ : The number 3 is prime.
$q$ : The number 3 is odd.
$p$ and $q$ both are true.

(ii) $p$ : All integers are positive.
$q$ : All integers are negative.
$p$ and $q$ both are false.

**7.** (i) Given, $\lim\limits_{x\to a}\dfrac{x^5-a^5}{x-a}=405$

$\Rightarrow\ 5(a)^{5-1}=405 \qquad \therefore\ \ 5a^4=405$

$\Rightarrow\ a^4=81\Rightarrow a=\pm 3$

(ii) Let $f(x)=e^x\Rightarrow f(x+h)=e^{x+h}$

Then, $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$\Rightarrow\quad f'(x)=\lim\limits_{h\to 0}\dfrac{e^{x+h}-e^x}{h}=\lim\limits_{h\to 0}\dfrac{e^x\cdot e^h-e^x}{h}$

$=\lim\limits_{h\to 0}\dfrac{e^x(e^h-1)}{h}=e^x\lim\limits_{h\to 0}\dfrac{e^h-1}{h}=e^x\cdot 1$

$\left[\because\ \lim\limits_{h\to 0}\dfrac{e^h-1}{h}=1\right]$

Thus, $\dfrac{d}{dx}(e^x)=e^x$.

**8.** (i) (a) Compound statement with 'AND'
25 is a multiple of 5 and 8.
This is a false statement since $p$ and $q$ both are not true.

(b) Compound statement with 'OR'
25 is a multiple of 5 or it is a multiple of 8.
This is a true statement.

(ii) We have,
$f(x) = mx + c \Rightarrow f'(x) = m(1) + 0 = m$
Given $f(0) = 1 \Rightarrow m(0) + c = 1 \Rightarrow c = 1$
Also, given $f'(0) = 1 \Rightarrow m = 1$
Now $f(x) = mx + c \Rightarrow f(x) = x + 1$
$\Rightarrow\ f(2) = 2 + 1 = 3$

**9.** $\lim\limits_{x\to 0}\dfrac{\cos ax-\cos bx}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{-2\sin\left(\dfrac{a+b}{2}x\right)\sin\left(\dfrac{a-b}{2}x\right)}{x^2}$

$=-2\lim\limits_{x\to 0}\dfrac{\sin\left(\dfrac{a+b}{2}x\right)}{x}\lim\limits_{x\to 0}\dfrac{\sin\left(\dfrac{a-b}{2}x\right)}{x}$

$$= -2\lim_{x\to 0}\frac{\sin\left(\frac{a+b}{2}x\right)}{\frac{a+b}{2}x}\cdot\frac{a+b}{2}\lim_{x\to 0}\frac{\sin\left(\frac{a-b}{2}x\right)}{\frac{a-b}{2}x}\cdot\frac{a-b}{2}$$

$$= -2(1)\left(\frac{a+b}{2}\right)(1)\left(\frac{a-b}{2}\right) = \frac{b^2-a^2}{2}$$

**10.** (i) We have,

$$\frac{d}{dx}\left\{\frac{\sin(x-a)}{\cos x}\right\}$$

$$= \frac{d}{dx}\left\{\frac{\sin x\cos a - \cos x\sin a}{\cos x}\right\}$$

$$= \frac{d}{dx}\{\tan x\cos a - \sin a\}$$

$$= \frac{d}{dx}(\tan x\cos a) - \frac{d}{dx}(\sin a)$$

$$= \cos a\frac{d}{dx}(\tan x) - \frac{d}{dx}(\sin a)$$

$$= \cos a \times \sec^2 x - 0 = \sec^2 x\cos a$$

(ii) We have, $\frac{d}{dx}(x^2\tan x)$

$$= \left\{x^2\cdot\frac{d}{dx}(\tan x) + \tan x\cdot\frac{d}{dx}(x^2)\right\}$$

$$= x^2\sec^2 x + 2x\tan x$$

**11.** $$\lim_{x\to\frac{\pi}{6}}\frac{\sqrt{3}\sin x - \cos x}{\left(x-\frac{\pi}{6}\right)}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{2\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right)}{\left(x-\frac{\pi}{6}\right)}$$

$$= \lim_{x\to\frac{\pi}{6}}\frac{2\left(\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}\right)}{\left(x-\frac{\pi}{6}\right)}$$

$$= 2\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\left(x-\frac{\pi}{6}\right)} = 2\lim_{y\to 0}\frac{\sin y}{y} = 2$$

$$\left[\text{putting}\left(x-\frac{\pi}{6}\right) = y\right]$$

**12.** (i) The negation of the statement of "The number $x$ is not a rational number" is : The number $x$ is a rational number.

The second statement is same : "The number $x$ is not an irrational number". Therefore, the given statements are negation of each other.

(ii) The negation of the statement of "The number $x$ is not a rational number" is : The number $x$ is a rational number.

The second statement is not same : "The number $x$ is an irrational number". Therefore, the given statements are not negation of each other.

(iii) The negation of the statement of "Two lines are not parallel" is : The lines are parallel.

The second statement is same : "Two lines does not intersect at a point".

Therefore, the given statements are negation of each other.

**13.** (i) Let $p$ be the statement given by

$p : \sqrt{3}$ is irrational.

If possible, let $p$ be not true. Then, $p$ is false.

$\Rightarrow$ $\sqrt{3}$ is rational

$\Rightarrow \sqrt{3} = \frac{a}{b}$, where $a$ and $b$ are integers having no common factor.

$\Rightarrow 3 = \frac{a^2}{b^2} \Rightarrow a^2 = 3b^2$

$\Rightarrow 3$ divides $a^2 \Rightarrow 3$ divides $a$

$\Rightarrow a = 3c$ for some integer $c$

$\Rightarrow a^2 = 9c^2 \Rightarrow 3b^2 = 9c^2 \Rightarrow b^2 = 3c^2$

$\Rightarrow 3$ divides $b^2 \Rightarrow 3$ divides $b$

Hence, our supposition was wrong so, $\sqrt{3}$ is irrational.

(ii) We have,

$$(\text{L.H.L. of } f(x) \text{ at } x = 1) = \lim_{x\to 1^-} f(x)$$

$$= \lim_{h\to 0} f(1-h) = \lim_{h\to 0} 5(1-h) - 4 = \lim_{h\to 0}(1-5h) = 1$$

$$(\text{R.H.L. of } f(x) \text{ at } x = 1) = \lim_{x\to 1^+} f(x)$$

$$= \lim_{h\to 0} f(1+h) = \lim_{h\to 0} 4(1+h)^3 - 3(1+h)$$

$$= 4(1)^3 - 3(1) = 1$$

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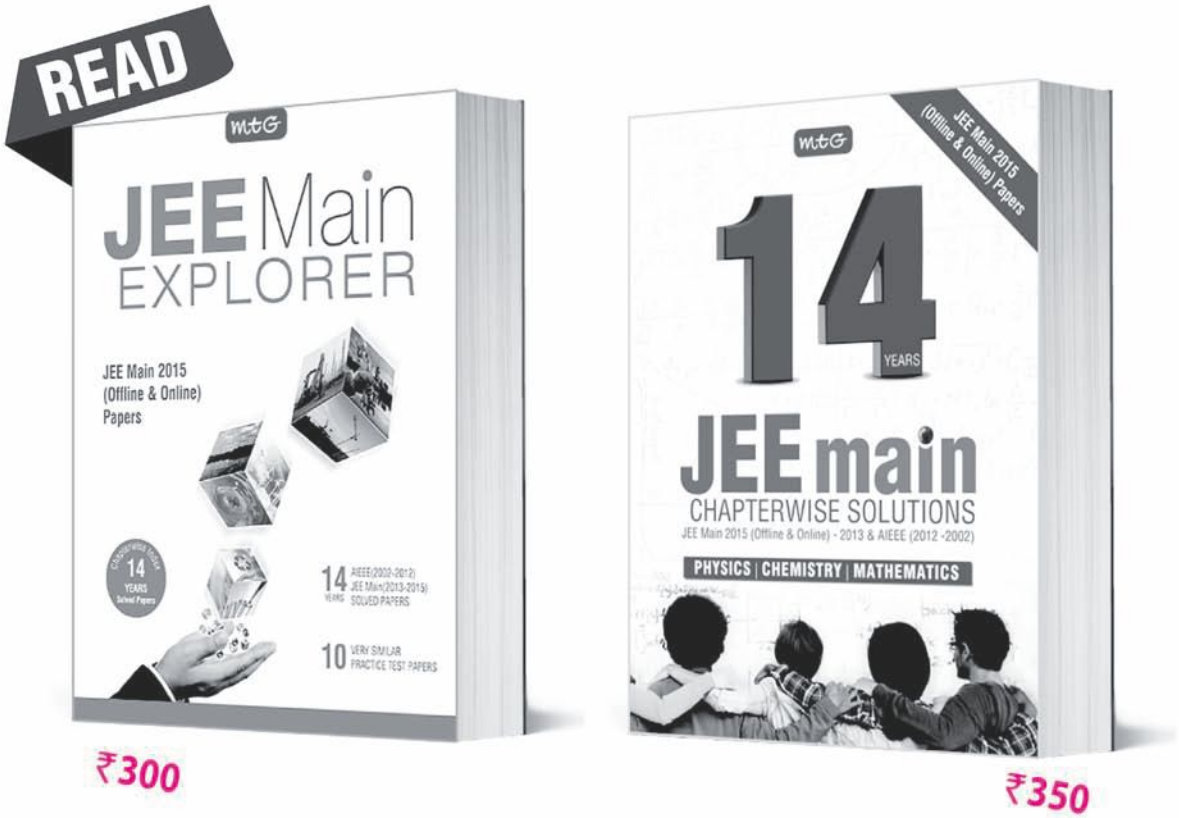


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Clearly, $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$

So, $\lim_{x\to1} f(x)$ exists and is equal to 1.

**14.** We have,

$$\lim_{x\to0}\frac{\tan x+4\tan 2x-3\tan 3x}{x^2\tan x}$$

$$=\lim_{x\to0}\frac{\tan x+4\dfrac{2\tan x}{1-\tan^2 x}-3\left(\dfrac{3\tan x-\tan^3 x}{1-3\tan^2 x}\right)}{x^2\tan x}$$

$$=\lim_{x\to0}\frac{1+\dfrac{8}{1-\tan^2 x}-3\left(\dfrac{3-\tan^2 x}{1-3\tan^2 x}\right)}{x^2}$$

$$=\lim_{x\to0}\frac{(1-\tan^2 x)(1-3\tan^2 x)+8(1-3\tan^2 x)}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$=\lim_{x\to0}\frac{-3(3-\tan^2 x)(1-\tan^2 x)}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$=\lim_{x\to0}\frac{1-4\tan^2 x+3\tan^4 x+8-24\tan^2 x-9}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$=\lim_{x\to0}\frac{+12\tan^2 x-3\tan^4 x}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$=\lim_{x\to0}\frac{-16\tan^2 x}{x^2(1-\tan^2 x)(1-3\tan^2 x)}$$

$$=-16\lim_{x\to0}\left(\frac{\tan x}{x}\right)^2\times\lim_{x\to0}\left(\frac{1}{1-\tan^2 x}\right)$$

$$\times\lim_{x\to0}\left(\frac{1}{1-3\tan^2 x}\right)=-16$$

**15.** Let $f(x) = x^2\cos x$. Then, $f(x+h) = (x+h)^2\cos(x+h)$

$$\frac{d}{dx}(f(x))=\lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=\lim_{h\to0}\frac{(x+h)^2\cos(x+h)-x^2\cos x}{h}$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=\lim_{h\to0}\frac{(x^2+2hx+h^2)\cos(x+h)-x^2\cos x}{h}$$

$$\{x^2\cos(x+h)-x^2\cos x\}$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=\lim_{h\to0}\frac{+2hx\cos(x+h)+h^2\cos(x+h)}{h}$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=\lim_{h\to0}[x^2\left\{\frac{\cos(x+h)-\cos x}{h}\right\}$$

$$+2x\cos(x+h)+h\cos(x+h)]$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=\lim_{h\to0}-2x^2\frac{\sin\left(x+\dfrac{h}{2}\right)\sin\dfrac{h}{2}}{h}$$

$$+\lim_{h\to0}2x\cos(x+h)+\lim_{h\to0}h\cos(x+h)$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=-x^2\lim_{h\to0}\frac{\sin\left(x+\dfrac{h}{2}\right)\sin\dfrac{h}{2}}{\dfrac{h}{2}}$$

$$+\lim_{h\to0}2x\cos(x+h)+\lim_{h\to0}h\cos(x+h)$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=-x^2\sin x+2x\cos x+(0\times\cos x)$$

$$\Rightarrow\quad \frac{d}{dx}(f(x))=-x^2\sin x+2x\cos x$$

**16.** (i) We have $y = x\tan x$ ...(1)

$$\Rightarrow\quad \frac{dy}{dx}=x\frac{d}{dx}(\tan x)+\tan x\frac{d}{dx}(x)$$

$$= x\sec^2 x + \tan x\cdot 1 = x\sec^2 x + \tan x$$

$$\therefore\quad x\sin^2 x\frac{dy}{dx}=x\sin^2 x(x\sec^2 x+\tan x)$$

$$=x^2\sin^2 x\cdot\frac{1}{\cos^2 x}+x\tan x\sin^2 x$$

$$= x^2\tan^2 x + x\tan x\sin^2 x = y^2 + y\sin^2 x$$

[Using (1)]

(ii) We have, $y=\dfrac{x}{x+5}$

Then

$$\frac{dy}{dx}=\frac{(x+5)1-x(1)}{(x+5)^2}=\frac{x+5-x}{(x+5)^2}=\frac{5}{(x+5)^2}$$

Now L.H.S. $= x\dfrac{dy}{dx}=\dfrac{5x}{(x+5)^2}$ ...(1)

R.H.S. $= y(1-y)=\dfrac{x}{x+5}\left(1-\dfrac{x}{x+5}\right)$

$$=\frac{x}{x+5}\cdot\frac{x+5-x}{x+5}=\frac{5x}{(x+5)^2}$$ ...(2)

From (1) and (2),

$$x\frac{dy}{dx}=y(1-y)$$

**17.** The necessary and sufficient condition that the integer $m$ be odd is $m^3$ must be odd. Let $p$ and $q$ denote the statements.

$p$ : The integer $m$ is odd.
$q$ : $m^3$ is odd.

To check the validity of "$p$ if $q$", we have to check whether "if $p$ then $q$" and "if $q$ then $p$" is true.

**Case 1 :** If $p$, then $q$

If $p$, then $q$ is the statement.

If the integer $m$ is odd, then $m^3$ is odd. We have to check whether this statement is true. Let us assume that $m$ is odd. The $m = 2k + 1$, where $k$ is an integer. Thus

$m^3 = (2k + 1)^3$

$= 8k^3 + 1 + 6k(2k + 1) = 8k^3 + 12k^2 + 6k + 1$

Therefore, $m^3$ is one more than an even number and hence is odd.

**Case 2 :** If $q$, then $p$

If $q$, then $p$ is the statement.

If $m$ is an integer and $m^3$ is odd, then $m$ is odd.

We have to check whether this statement is true. We check this by contrapositive method. The contrapositive of the given statement is :

If $m$ is an even integer, then $m^3$ is an even integer.

$m$ is even implies that $m = 2k$ for some $k$. Then $m^3 = 8k^3$. Therefore, $m^3$ is even.

Thus, the given statement is true.

**18.** (i) $\lim\limits_{x\to2}\left(\dfrac{x^8-16}{x^4-4}+\dfrac{x^2-9}{x-3}\right)=\lim\limits_{x\to2}\dfrac{x^8-16}{x^4-4}+\lim\limits_{x\to2}\dfrac{x^2-9}{x-3}$

$=\lim\limits_{x\to2}\dfrac{(x^4+4)(x^4-4)}{x^4-4}+\lim\limits_{x\to2}\dfrac{(x+3)(x-3)}{x-3}$

$=\lim\limits_{x\to2}(x^4+4)+\lim\limits_{x\to2}(x+3)$

$= (2^4 + 4) + (2 + 3) = 20 + 5 = 25$

(ii) Put $2x + 4 = y$

$\Rightarrow \quad x=\dfrac{y-4}{2}$

As $x \to 2$, $y \to 8$

$\therefore \quad \lim\limits_{x\to2}\dfrac{(2x+4)^{1/3}-2}{x-2}=\lim\limits_{y\to8}\dfrac{y^{1/3}-2}{\frac{y-4}{2}-2}$

$=2\lim\limits_{y\to8}\dfrac{y^{1/3}-8^{1/3}}{y-4-4}=2\lim\limits_{y\to8}\dfrac{y^{1/3}-8^{1/3}}{y-8}$

$=2\dfrac{1}{3}(8)^{(1/3)-1}=\dfrac{2}{3}(2^3)^{-2/3}=\dfrac{2}{3}\times\dfrac{1}{4}=\dfrac{1}{6}$


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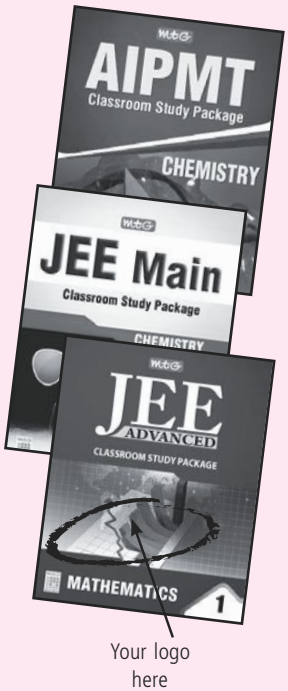


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# OLYMPIAD CORNER

**1.** In triangle $ABC$, the median $AD$ is the geometric mean of $AB$ and $AC$.
Prove that
$1 + \cos A = \sqrt{2}\ |\cos B - \cos C|$.

**2.** Let $r$ be the inradius of a tetrahedron $A_1A_2A_3A_4$, and let $r_1, r_2, r_3, r_4$ be the inradii of triangles $A_2A_3A_4$, $A_1A_3A_4$, $A_1A_2A_4$, $A_1A_2A_3$ respectively.
Prove that

$$\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}+\frac{1}{r_4^2}\le\frac{2}{r^2},$$

with equality if the tetrahedron is regular.

**3.** Let $a = \sin 10°$, $b = \sin 50°$, $c = \sin 70°$. Prove that
(i) $a + b = c$, (ii) $a^{-1} + b^{-1} = c^{-1} + 6$.

**4.** Find all polynomials $f(x) = x^n + a_1x^{n-1} + \ldots + a_n$ with the following properties:
(i) all the coefficients $a_1, a_2, \ldots, a_n$ belong to the set $\{-1,1\}$;
(ii) all the roots of the equation $f(x) = 0$ are real.

**5.** (a) Find all positive integers $p \le q \le r$ satisfying the equation
$p + q + r + pq + qr + rp = pqr + 1$.
(b) For each such solution $(p, q, r)$, evaluate $\tan^{-1}(1/p) + \tan^{-1}(1/q) + \tan^{-1}(1/r)$.

## SOLUTIONS

**1.** Let $a, b, c$ be the sides and $\alpha, \beta, \gamma$ be the angles of the triangle. We are given that

$$bc = (AD)^2 = \frac{b^2+c^2}{2}-\frac{a^2}{4},$$

or $a^2 = 2(b - c)^2$ …(1)

Without loss of generality we may assume $b > c$, so $\beta > \gamma$ and $\cos\beta < \cos\gamma$.

Then from (1), it follows that $a = \sqrt{2}\,(b - c)$
From the law of sines

$\sin\alpha = \sqrt{2}(\sin\beta - \sin\gamma)$ …(2)

We have to show

$1 + \cos\alpha = \sqrt{2}(\cos\gamma - \cos\beta)$ …(3)

From (2) and (3), it is sufficient to prove that

$$\frac{\sin\alpha}{1+\cos\alpha}=\frac{\sin\beta-\sin\gamma}{\cos\gamma-\cos\beta},$$

or $$\tan\frac{\alpha}{2}=\frac{2\sin[(\beta-\gamma)/2]\cos[(\beta+\gamma)/2]}{2\sin[(\beta-\gamma)/2]\sin[(\beta+\gamma)/2]}$$

$$=\cot\frac{\beta+\gamma}{2},$$ and that holds.

**2.** Let $S_1, S_2, S_3, S_4$ be the areas of the faces $A_2A_3A_4$, $A_1A_3A_4$, $A_1A_2A_4$, $A_1A_2A_3$; let $\alpha, \beta, \gamma$ be the dihedral angles at the edges $A_2A_3, A_2A_4, A_3A_4$; and let $h_1$ be the altitude from the vertex $A_1$ of the tetrahedron and $h_1' = A_1E$ be the altitude of the face $A_1A_2A_3$. Then

$$h_1 = h_1'\sin\alpha = \frac{2S_4}{A_2A_3}\sqrt{(1+\cos\alpha)(1-\cos\alpha)}$$

$$=\frac{2}{A_2A_3}\sqrt{(S_4+S_4\cos\alpha)(S_4-S_4\cos\alpha)}$$ …(1)

and analogously

$$h_1=\frac{2}{A_2A_4}\sqrt{(S_3+S_3\cos\beta)(S_3-S_3\cos\beta)},$$ …(2)

$$h_1=\frac{2}{A_3A_4}\sqrt{(S_2+S_2\cos\gamma)(S_2-S_2\cos\gamma)}$$ …(3)

From (1), (2) and (3), we get

$$h_1=\frac{2}{A_2A_3+A_2A_4+A_3A_4}.Q$$ …(4)

where

$$Q=\sqrt{(S_4+S_4\cos\alpha)(S_4-S_4\cos\alpha)}+\sqrt{(S_3+S_3\cos\beta)(S_3-S_3\cos\beta)}+\sqrt{(S_2+S_2\cos\gamma)(S_2-S_2\cos\gamma)}.$$

According to Cauchy's inequality, we have

$$Q\le\{(S_4+S_4\cos\alpha)+(S_3+S_3\cos\beta)+(S_2+S_2\cos\gamma)\}^{1/2}\cdot\{(S_4-S_4\cos\alpha)+(S_3-S_3\cos\beta)+(S_2-S_2\cos\gamma)\}^{1/2}$$

$$=\{S_4+S_3+S_2+(S_4\cos\alpha+S_3\cos\beta+S_2\cos\gamma)\}^{1/2}\cdot\{S_4+S_3+S_2-(S_4\cos\alpha+S_3\cos\beta+S_2\cos\gamma)\}^{1/2}$$

$$=(S_4+S_3+S_2+S_1)^{1/2}(S_4+S_3+S_2-S_1)^{1/2}$$

$$=S^{1/2}(S-2S_1)^{1/2} \quad \ldots(5)$$

where $S=S_1+S_2+S_3+S_4$. From (4) and (5), we get

$$h_1\le\frac{2\sqrt{S(S-2S_1)}}{A_2A_3+A_2A_4+A_3A_4}.$$

Therefore [since $rS=h_1S_1$ = three times the volume of the tetrahedron], we obtain

$$r=\frac{h_1S_1}{S}\le\frac{2S_1}{A_2A_3+A_2A_4+A_3A_4}.\frac{\sqrt{S(S-2S_1)}}{S}$$

$$=r_1\sqrt{\frac{S-2S_1}{S}}.$$

In the same manner, we have

$$r\le r_i\sqrt{\frac{S-2S_i}{S}},\ i=1,2,3,4$$

Hence

$$\sum_{i=1}^{4}\frac{1}{r_i^2}\le\frac{1}{r^2}\sum_{i=1}^{4}\frac{S-2S_i}{S}=\frac{2}{r^2}.$$

**3.** Since, $\sin 30^\circ=\sin 150^\circ=-\sin 210^\circ=\frac{1}{2}$,

we have

$$\sin(3\cdot10)^\circ=\sin(3\cdot50)^\circ=\sin(3\cdot-70)^\circ=\frac{1}{2} \quad \ldots(1)$$

But $\sin^3x=\frac{3}{4}\sin x-\frac{1}{4}\sin3x$,

so that by (1)

$$\sin^310^\circ=\frac{3}{4}\sin10^\circ-\frac{1}{8},$$

$$\sin^350^\circ=\frac{3}{4}\sin50^\circ-\frac{1}{8}$$

and $\sin^3(-70)^\circ=\frac{3}{4}\sin(-70)^\circ-\frac{1}{8}$,

*i.e.* $a, b, -c$ are the three roots of

$$f(t)=t^3-\frac{3}{4}t+\frac{1}{8}.$$

Therefore $a+b+(-c)=0$, …(i)

$$ab(-c)=\frac{1}{8} \quad \ldots(ii)$$

and $ab+b(-c)+(-c)a=-\frac{3}{4}$

Dividing by $abc$ and using (ii) leads to

$$c^{-1}-a^{-1}-b^{-1}=\left(-\frac{3}{4}\right)\cdot8 \quad \ldots(iii)$$

**4.** Let $r_1, r_2, \ldots, r_n$ represent the roots of $f(x)$. Then

$$\sum_{i=1}^{n}r_i^2=\left(\sum_{i=1}^{n}r_i\right)^2-2\sum_{i<j}r_ir_i=a_1^2-2a_2=1-2a_2 \quad \ldots(1)$$


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By the Arithmetic Mean-Geometric Mean inequality, we have

$$\frac{\sum r_i^2}{n} \geq \left(\prod r_i^2\right)^{1/n} = (a_n^2)^{1/n} = 1$$

with equality if and only if $r_i^2 = 1$ for all $i$

From (1), we obtain $1 - 2a_2 \geq n$

This implies

1. $n \leq 3$
2. If $n = 3$, $r_i = \pm 1$ for all $i$
3. If $n = 2, 3$, $a_2 = -1$

This gives the following polynomials:

$n = 1$ $f(x) = x + 1$ or $f(x) = x - 1$

$n = 2$ $f(x) = x^2 + x - 1$ or $f(x) = x^2 - x - 1$

$n = 3$ $f(x) = (x - 1)^2(x + 1) = x^3 + x^2 - x - 1$

or $f(x) = (x + 1)^2(x - 1) = x^3 - x^2 - x + 1$

The set of polynomials is

$\{x - 1, x + 1, x^2 - x - 1, x^2 + x - 1, x^3 + x^2 - x - 1, x^3 - x^2 - x + 1\}$.

**5.** (a) There are exactly 3 solutions, given by $(p, q, r) = (2, 4, 13), (2, 5, 8)$ and $(3, 3, 7)$.

Note first that

$(p - 1)(q - 1)(r - 1) = pqr - (pq + qr + rp) + (p + q + r) - 1$

which becomes, using the given equation,

$(p - 1)(q - 1)(r - 1) = 2(p + q + r - 1)$ ...(1)

If $p \geq 4$, then $4 \leq p \leq q \leq r$ implies

$(p - 1)(q - 1)(r - 1) \geq 9(r - 1)$

and $2(p + q + r - 1) \leq 2(3r - 1)$;

and since

$9(r - 1) - 2(3r - 1) = 3r - 7 > 0$,

(1) cannot hold in this case. Thus $p < 4$. Since $p = 1$ clearly does not satisfy (1), we have $p = 2$ or 3. When $p = 2$, (1) becomes $(q - 1)(r - 1) = 2(q + r + 1)$ or $(q - 3)(r - 3) = 10$. Thus $q - 3 = 1$, $r - 3 = 10$ or $q - 3 = 2$, $r - 3 = 5$, yielding two solutions: $(2, 4, 13)$ and $(2, 5, 8)$. When $p = 3$, (1) becomes $(q - 1)(r - 1) = q + r + 2$ or $(q - 2)(r - 2) = 5$ which yields the third solution: $(3, 3, 7)$.

(b) The value is $\pi/4$ in all cases. To see this, set $A = \tan^{-1}(1/p)$, $B = \tan^{-1}(1/q)$ and $C = \tan^{-1}(1/r)$. Since

$$0 < \frac{1}{r} \leq \frac{1}{q} \leq \frac{1}{p} < 1,$$

we have $0 < C \leq B \leq A < \pi/4$ and thus

$0 < A + B + C < 3\pi/4$.

From the well know formula for $\tan(x + y)$ one easily deduces that for all $x, y, z$ with

$x + y + z \neq k\pi + \pi/2$ (where $k$ denotes an integer).

$$\tan(x + y + z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - (\tan x \tan y + \tan y \tan z + \tan z \tan x)}$$

Thus

$$\tan(A + B + C) = \frac{\frac{1}{p} + \frac{1}{q} + \frac{1}{r} - \frac{1}{pqr}}{1 - \left(\frac{1}{pq} + \frac{1}{qr} + \frac{1}{rp}\right)}$$

$$= \frac{pq + qr + rp - 1}{pqr - (p + q + r)} = 1$$

Hence $A + B + C = \pi/4$, that is

$$\tan^{-1}(1/p) + \tan^{-1}(1/q) + \tan^{-1}(1/r) = \frac{\pi}{4}.$$


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# QUANTITATIVE APTITUDE TEST

Useful for Bank PO, Specialist Officers & Clerical Cadre, BCA, MAT, CSAT, CDS and other such examinations.

**1.** A sum of money is to be distributed among $A$, $B$, $C$ and $D$ in the proportion of 5 : 2 : 4 : 3. If $C$ gets ₹ 1000 more than $D$, what is $B$'s share?
(a) ₹ 500 (b) ₹ 1500
(c) ₹ 2000 (d) None of these

**2.** A shopkeeper sold an article offering a discount of 5% and earned a profit of 23.5%. What would have been the percentage of profit earned if no discount was offered?
(a) 24.5 (b) 28.5
(c) 30 (d) None of these

**3.** $A$ and $B$ can do a piece of work in 45 days and 40 days respectively. They began to do the work together but $A$ leaves after some days and then $B$ completed the remaining work in 23 days. The number of days after which $A$ left the work was
(a) 6 (b) 8 (c) 9 (d) 12

**4.** $A$, $B$ and $C$ jointly thought of engaging themselves in a business venture. It was agreed that $A$ would invest ₹ 6500 for 6 months, $B$, ₹ 8400 for 5 months and $C$, ₹ 10000 for 3 months. A wants to be the working member for which he was to receive 5% of the profits. The profit earned was ₹ 7400. Calculate the share of $B$ in the profit.
(a) ₹ 1900 (b) ₹ 2660
(c) ₹ 2800 (d) ₹ 2840

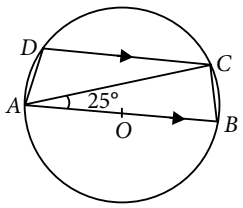
**5.** If one of the interior angles of a regular polygon is equal to $\frac{9}{8}$ of each interior angle of a regular hexagon, then the number of sides of the polygon is
(a) 4 (b) 5 (c) 7 (d) 8

**6.** A train passes two persons walking in the same direction in which the train is going. These persons are walking at the rate of 3 km/hr and 5 km/hr respectively and the train passes them completely in 10 seconds and 11 seconds respectively. The speed of the train is
(a) 24 km/hr (b) 25 km/hr
(c) 27 km/hr (d) 28 km/hr

**7.** A man arranges to pay off a debt of ₹ 3600 by 40 annual installments which are in A.P. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid. The value of the $8^{th}$ installment is
(a) ₹ 35 (b) ₹ 50
(c) ₹ 65 (d) None of these

**8.** The difference between the radii of the smaller circle and the bigger circle is 7 cm and the difference between the areas of the two circles is 1078 $cm^2$. Radius of the smaller circle is
(a) 17.5 cm (b) 21 cm
(c) 28 cm (d) None of these

**9.** In the given figure, $AOB$ is a diameter of the circle and $CD \parallel AB$. If $\angle CAB = 25°$, then $\angle CAD =$




(a) 45° (b) 40° (c) 65° (d) 115°

**10.** $(2 \sin^2 30° - 3 \cos^2 45° + \tan^2 60°) =$
(a) 1 (b) 2 (c) 4 (d) 5

**11.** $999\frac{1}{7} + 999\frac{2}{7} + 999\frac{3}{7} + 999\frac{4}{7} + 999\frac{5}{7} + 999\frac{6}{7} =$
(a) 2997 (b) 5979
(c) 5997 (d) 5994

**12.** Simple interest on a certain sum at a certain annual rate of interest is $\frac{25}{16}$ of the sum. If the numbers representing rate per cent and time in years be equal, then rate per cent per annum is

(a) 8% (b) $11\frac{1}{2}\%$

(c) $12\frac{1}{4}\%$ (d) $12\frac{1}{2}\%$

**13.** $\sqrt{\frac{(0.27)^2+(0.21)^2+(0.29)^2}{(0.027)^2+(0.021)^2+(0.029)^2}} =$

(a) 0.1 (b) 0.01 (c) 10 (d) 100

**14.** A covered wooden box has the inner measures as 115 cm, 75 cm and 35 cm and the thickness of wood is 2.5 cm. The volume of the wood is

(a) 81000 cu. cm (b) 81775 cu. cm

(c) 82125 cu. cm (d) None of these

**15.** The sum of the roots of the equation $5x^2 + (p + q + r)x + pqr = 0$, is equal to zero. What is the value of $(p^3 + q^3 + r^3)$ ?

(a) $2pqr$ (b) $3pqr$ (c) $4pqr$ (d) $5pqr$

**16.** The points $A(1, 2)$, $B(5, 4)$, $C(3, 8)$ and $D(-1, 6)$ taken in order are the vertices of a

(a) rectangle (b) rhombus

(c) square (d) None of these

**17.** If $(1^2 + 2^2 + 3^2 + .... + x^2) = \frac{x(x+1)(2x+1)}{6}$, then $(1^2 + 3^2 + 5^2 + .... + 19^2) =$

(a) 1330 (b) 2100 (c) 2485 (d) 2500

**18.** If $\log_{10} 2 = 0.3010$ and $\log_{10} 7 = 0.8451$, then the value of $\log_{10} 2.8$ is

(a) 0.4471 (b) 1.4471

(c) 2.4471 (d) None of these

**19.** A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45° with the man's eye when at a distance of 60 metres from the tower. After 5 seconds, the angle of depression becomes 30°. What is the approximate speed of the boat, assuming that it is running in still water?

(a) 32 km/hr (b) 36 km/hr

(c) 38 km/hr (d) 40 km/hr

**20.** Three numbers are in the ratio 1 : 2 : 3 and their HCF is 12. The numbers are

(a) 4, 8, 12 (b) 5, 10, 15

(c) 10, 20, 30 (d) 12, 24, 36

**21.** Find the ratio in which rice at ₹ 7.20/kg be mixed with rice at ₹ 5.7/kg to produce a mixture worth ₹ 6.3/kg.

(a) 1 : 3 (b) 2 : 3

(c) 3 : 4 (d) 4 : 5

**22.** A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

(a) 32 (b) 48 (c) 64 (d) 96

**23.** If $a + b + c = 0$, then find the value of

$$\frac{a^4+b^4+c^4}{b^2c^2+c^2a^2+a^2b^2}$$

(a) 0 (b) 1 (c) 2 (d) 3

**24.** A bag contains 4 white, 5 red and 6 blue balls. Three balls are drawn at random from the bag. The probability that all of them are red, is

(a) $\frac{1}{22}$ (b) $\frac{3}{22}$ (c) $\frac{2}{91}$ (d) $\frac{2}{77}$

**25.** Three years ago, the age of a father was 7 times that of his son. Three years hence the father's age would be four times that of his son. The present ages of the father and his son respectively are

(a) 45, 9 (b) 40, 5

(c) 50, 10 (d) 45, 8

## ANSWER KEYS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (c) | **2.** | (c) | **3.** | (c) | **4.** | (b) | **5.** | (d) |
| **6.** | (b) | **7.** | (c) | **8.** | (b) | **9.** | (b) | **10.** | (b) |
| **11.** | (c) | **12.** | (d) | **13.** | (c) | **14.** | (c) | **15.** | (b) |
| **16.** | (c) | **17.** | (a) | **18.** | (a) | **19.** | (a) | **20.** | (d) |
| **21.** | (b) | **22.** | (c) | **23.** | (c) | **24.** | (c) | **25.** | (a) |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# CONCEPT BOOSTERS

Class XI

## PERMUTATION AND COMBINATION

*ALOK KUMAR, B.Tech, IIT Kanpur

This column is aimed at Class XI students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

### INTRODUCTION

The present chapter can be given the name as counting techniques or counting without actual counting. For example if we are interested in counting the number of 9 digit numbers which are divisible by 9, then it will take a lot of time to count the number of such numbers, but by using the techniques given in this chapter, this will be easy enough to find the number.

The basis of all the techniques given in this chapter are based on two fundamental principles of counting namely addition and multiplication principles. Using these principles, formulae for number of ways of arrangements and selections of different objects are derived.

### FUNDAMENTAL PRINCIPLE OF COUNTING

**Multiplication Principle:** If an operation can be performed in '$m$' different ways following which a second operation can be performed in '$n$' different ways, then the two operations in succession can be performed in $m \times n$ ways. This can be extended to any finite number of operations.

### PERMUTATION

**Definition :** Each of the different arrangements which can be made by taking some (or all) of a number of given things is called permutation.

**Note :**

Factorial Notation : The continued product of first $n$ natural numbers is generally written as $n!$ and is read as factorial $n$ *i.e.*,

$n! = n\,(n-1)(n-2)\ldots 3.\,2.\,1$

- **Some Important Properties**

**(i)** $n! = n\,(n-1)\,!$

**(ii)** $(2n)! = 2^n \cdot n!\,[1 \cdot 3 \cdot 5 \cdot 7 \ldots \cdot (2n-1)]$

**(iii)** $0! = 1! = 1$

**(iv)** factorial of negative integers are not defined

- **Arrangement of $n$ different things**

The number of permutations of $n$ different things taken all at a time $= {}^nP_n = n!$

- **Meaning and value of ${}^nP_r$**

The number of permutations of $n$ different things, taken $r$ at a time is denoted by ${}^nP_r$ or $P\,(n, r)$.

$${}^nP_r = \frac{n!}{(n-r)!}(0 \le r \le n)$$

$$= n\,(n-1)\,(n-2)\ldots(n-r+1).$$

**Note :**

**(i)** ${}^nP_0 = 1$, ${}^nP_1 = n$ and ${}^nP_{n-1} = {}^nP_n = n!$.

**(ii)** ${}^nP_r = n\,({}^{n-1}P_{r-1}) = n\,(n-1)\,({}^{n-2}P_{r-2})$

$= n\,(n-1)\,(n-2)\,({}^{n-3}P_{r-3}) = \ldots$

**Alternative :**

(Combinational proof) ${}^nP_r$ denotes the number of ways of arranging $r$-objects out of $n$-objects in a line. This work can be done in the following way also.

Suppose the objects are $a_1, a_2, \ldots, a_n$. First we find the number of permutations, in which $a_1$ does not appear. Number of such permutations is ${}^{n-1}P_r$. Further we consider those arrangements, in which $a_1$ necessarily appears. Number of such permutation is $r \cdot {}^{n-1}P_{r-1}$, as we can arrange $(r-1)$ objects out of $(n-1)$ objects in ${}^{n-1}P_{r-1}$

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ways and then in any such permutation we can fix the position of $a_1$ in $r$-ways.

Now using the principle of addition, the required number is ${}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$.

- **Arrangement when repetition is allowed**

The number of permutations of $n$ different things taken $r$ at a time, when each thing may be repeated any number of times, is $n^r$.

## ARRANGEMENT OF THINGS WHEN ALL ARE NOT DISTINCT

The number of permutations of $n$ things taken all at a time, $p$ are alike of one kind, $q$ are alike of second kind and $r$ are alike of a third kind and the rest $n - (p + q + r)$ are all different is $\frac{n!}{p!\,q!\,r!}$

- **Some Cases**

**(a)** Arrangement when certain things occur together.

**(b)** Arrangement when certain things do not occur together.

**Method :**

Required number of ways = Total ways (without restriction) - Number of ways in which they sit together.

**(c)** Arrangement when no two of a certain things occur together.

**(d)** Arrangement when order should not change.

If we want to arrange $ABC$ in such a way so that order of $A$ and $C$ should not change, it means we want those arrangement in which $A$ comes before $C$ *i.e.*,

$ABC$, $BAC$, $ACB$

For this we take $A$ & $C$ as identical objects and then $ABC$ can be arranged in 3!/2! = 3 ways.

**(e)** Rank of the word in the dictionary.

**(f)** Sum of numbers formed with the given digits.

**Method:** Suppose we have to find sum of numbers of four digits that can be formed with the given digit, first find the sum at unit place, ten's place, hundred's place & thousand's place. Sum of numbers = (sum at unit place) × 1 + (sum at ten's place) × 10 + (sum at hundred's place) × 100 + (sum at thousand's place) × 1000.

## COMBINATION (SELECTION)

**Definition :** Each of the different group or selection which can be made by taking some (or all) number of given things, without reference to the order of the things in each group, is called a combination.

**Meaning and value of ${}^nC_r$**

The number of combinations of $n$ different things taken $r$ at a time is denoted by

${}^nC_r$ or $C(n, r)$ or $\binom{n}{r}$

$${}^nC_r = \frac{n!}{r!(n-r)!} (0 \le r \le n)$$

$$= \frac{{}^nP_r}{r!} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots 2\cdot 1}$$

If $r > n$, then ${}^nC_r = 0$.

**Note :**

**(i)** ${}^nC_r$ is a natural number.

**(ii)** ${}^nC_0 = {}^nC_n = 1$, ${}^nC_1 = n$

**(iii)** ${}^nC_r = {}^nC_{n-r}$

**(iv)** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

**(v)** ${}^nC_x = {}^nC_y \Rightarrow x = y$ or $x + y = n$

**(vi)** $n \cdot {}^{n-1}C_{r-1} = (n - r + 1) \cdot {}^nC_{r-1}$

**(vii)** If $n$ is even then the greatest value of ${}^nC_r$ is ${}^nC_{n/2}$.

**(viii)** If $n$ is odd then the greatest value of ${}^nC_r$ is ${}^nC_{\frac{n+1}{2}}$ or ${}^nC_{\frac{n-1}{2}}$.

**(ix)** ${}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1}$

- **Selection from Distinct objects**

The number of ways (or combinations) of $n$ different things selecting at least one of them is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \ldots + {}^nC_n = 2^n - 1$.

- **Selection from Identical Objects**

1. The number of ways of selecting $r$ ($r \le n$) objects out of $n$ identical objects is 1.
2. The number of ways of selecting at least one object out of $n$ identical objects is $n$.

- **Selection when both identical & distinct objects are present**

If out of $(p + q + r + t)$ things, $p$ are alike of one kind, $q$ are alike of second kind, $r$ are alike of third kind and $t$ are different, then the total number of combinations is $(p + 1)(q + 1)(r + 1)2^t - 1$.

**(a) Total number of divisors of a given natural number**

Every natural number $N$ can always be put in the form $N = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$ where $p_1, p_2, \ldots, p_k$ are distinct primes and $a_1, a_2, \ldots a_k$ are non negative integers.

If $N = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$, then number of divisors of $N$ are $(a_1 + 1)(a_2 + 1) \ldots (a_k + 1)$ which includes 1 and $N$ also.

**Note :** All the divisors excluding 1 and $N$ are called proper divisors.

**(b) Sum of divisors**

The sum of the divisors of $N$

$= (1 + p_1 + p_1^2 + \ldots + p_1^{a_1})(1 + p_2 + p_2^2 + \ldots + p_2^{a_2}) \ldots (1 + p_k + p_k^2 + \ldots + p_k^{a_k})$

$$= \frac{p_1^{a_1+1} - 1}{p_1 - 1} \cdot \frac{p_2^{a_2+1} - 1}{p_2 - 1} \cdots \frac{p_k^{a_k+1} - 1}{p_k - 1}$$

**(c) Number of ways in which $N$ can be resolved as a product of two factors**

The number of ways of putting $N$ as a product of two natural numbers is

$\frac{1}{2}(a_1 + 1)(a_2 + 1) \ldots (a_k + 1)$ if $N$ is not a perfect square. If $N$ is a perfect square then this is $\frac{1}{2}[(a_1 + 1)(a_2 + 1) \ldots (a_k + 1) + 1]$.

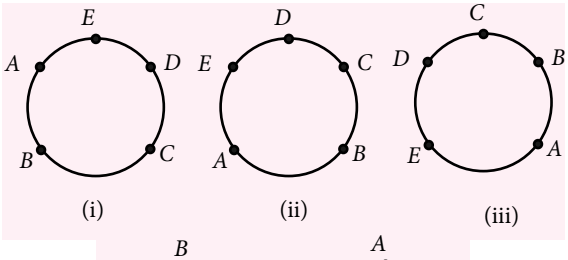
**(d) Number of ways in which a composite number $N$ can be resolved into two factors which are relatively prime or co-prime.**

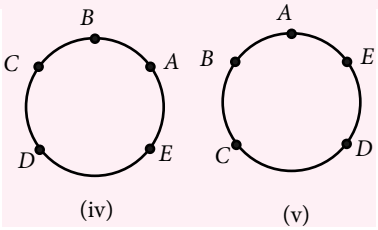
Number of ways $= 2^{n-1}$ where $n$ is the number of different prime factors in $N$.

## CIRCULAR PERMUTATIONS

- **Arrangements round a circular table :**

Consider five persons $A$, $B$, $C$, $D$ and $E$ to be seated on the circumference of a circular table in order (which has no head). Now, shifting $A$, $B$, $C$, $D$ and $E$ one position in anticlockwise direction we will get arrangements as follows:



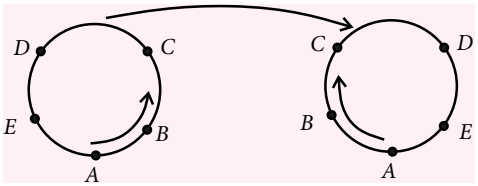




We see that arrangements in all figures are same.

$\therefore$ The number of circular permutations of $n$ different things taken all at a time is $\frac{{}^nP_n}{n} = (n - 1)!$, if clockwise and anticlockwise orders are taken as different.

- **Formation of necklace/garland using beads/ flowers (Ring permutation)**

Consider five beads $A$, $B$, $C$, $D$ and $E$ in a necklace or five flowers $A$, $B$, $C$, $D$ and $E$ in a garland etc. If the necklace or garland on the left is turned over we obtain the arrangement on the right, *i.e.*, anticlockwise and clockwise order of arrangements are not different.




Thus, the number of circular permutations of '$n$' different things taken all at a time is $\frac{1}{2}(n - 1)!$, if clockwise and anticlockwise orders are taken as not different.

- **Number of circular permutations of '$n$' different things taken '$r$' at a time.**

**Case I:** If clockwise and anticlockwise orders are taken as different, then the required number of circular permutation $= ({}^nP_r) / r$.

**Case II:** If clockwise and anticlockwise orders are taken as same, then the required number of circular permutations $= ({}^nP_r) / (2r)$

## DIVISION INTO GROUPS

- **Different number of objects in each group**

The number of ways in which $n$ distinct objects can be split into $r$ groups containing respectively $s_1, s_2, \ldots, s_r$ objects, where $s_i \neq s_j$, for $i \neq j$, is given by $\frac{n!}{s_1! s_2! \ldots s_r!}$.

- **Same number of objects in some group**

If $k$ of the numbers among $s_1, s_2, \ldots, s_r$ are equal, then the required number of ways is $\frac{n!}{s_1! s_2! \ldots s_r!\, k!}$

For example if 36 distinct objects are to be divided among 9 groups such that four groups have 2 objects each, three groups have 5 objects each and remaining two groups having 6 and 7 objects, then the required number of ways

$$= \frac{36!}{(2!)^4 \cdot 4!(5!)^3 \cdot 3! \cdot 6! \cdot 7!}.$$

## DISTRIBUTION AMONG PERSONS

The number of ways in which $n$ distinct objects can be distributed among $r$ persons in a required way = number of ways of dividing $n$ distinct objects in $r$-groups in the required way $\times$ $r!$.

For example if 36 distinct objects are to be divided among 9 persons such that four of the persons are getting 2 objects each, three of the persons are getting 5 objects each and the remaining two persons are getting 6 and 7 objects, then the number of ways of doing so is

$$\frac{36!}{(2!)^4 \cdot 4!(5!)^3 \cdot 3! \cdot 6! \cdot 7!} \times 9!.$$

## EXPONENT OF PRIME NUMBER ($p$) IN $n!$

Let $p$ be a given prime and $n$ is any positive integer, then maximum power of $p$ present in $n!$ is

$\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots$, where $[\cdot]$ denotes the greatest integer function. The proof of the above formula can be obtained using the fact that $\left[\frac{n}{m}\right]$ gives the number of integral multiples of $m$ in 1, 2, ... , $n$; for any positive integers $n$ and $m$. The above formula does not work for composite numbers. For example, if we have to find the maximum power of 6, present in 32!, then the answer is not $\left[\frac{32}{6}\right]+\left[\frac{32}{6^2}\right]+\ldots=5$, as 5 is the number of integral multiples of 6 in 1, 2, ... , 32; and 6 can be obtained on multiplying 2 by 3 also. Hence for the required number, we find the maximum powers of 2 and 3 (say $r$ and $s$) present in 32!. Using the above formula $r = 31$ and $s = 14$. Hence 2 and 3 will be combined (to form 6) 14 times. Thus maximum power of 6 present in 32! is 14.

## PRINCIPLE OF INCLUSION AND EXCLUSION

If $A_1, A_2, \ldots, A_m$ are $m$ sets and $n(S)$ denotes the number of elements in the set $S$, then

$$n\left(\bigcup_{k=1}^{m} A_k\right)=\sum_{k=1}^{m} n(A_k)-\sum_{1\le i<j\le m} n(A_i\cap A_j)+\ldots$$
$$+(-1)^{s-1}\sum_{i<i_1<i_2\ldots<i_s\le m} n\left(\bigcap_{k=1}^{s} A_{i_k}\right)+\ldots$$
$$+(-1)^{m-1} n\left(\bigcap_{k=1}^{m} A_k\right)$$

Note that if $x\in\bigcup_{k=1}^{m} A_k$, then $x$ belongs to atleast one of $A_k$, $1 \le k \le m$.

## DE-ARRANGEMENT

As another application of the principle of inclusion and exclusion, number of dearrangement of $n$ objects (number of ways in which $n$ numbered balls (from 1 to $n$) can be placed in $n$ numbered boxes (from 1 to $n$), one in each box, so that no ball goes to its corresponding numbered box) is given by $n!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\ldots+\frac{(-1)^n}{n!}\right)$.

Also $D_n = (n - 1)\,[D_{n-1} + D_{n-2}]$

## MULTINOMIAL THEOREM AND ITS APPLICATION

Consider the equation $x_1 + x_2 + \ldots + x_r = n$, where $a_i \le x_i \le b_i$ ; $a_i, b_i, x_i \in I$ ; $i = 1, 2, \ldots r$. In order to find the number of solutions of the given equation satisfying the given conditions we observe that the number of solutions is the same as the coefficient of $x^n$ in the product

$$(x^{a_1}+x^{a_1+1}+x^{a_1+2}+\ldots+x^{b_1})\times$$
$$(x^{a_2}+x^{a_2+1}+x^{a_2+2}+\ldots+x^{b_2})\times$$
$$(x^{a_3}+x^{a_3+1}+x^{a_3+2}+\ldots+x^{b_3})\times\cdots$$
$$\ldots\times(x^{a_r}+x^{a_r+1}+x^{a_r+2}+\ldots+x^{b_r})$$

For example, if we have to find the number of non negative integral solutions of $x_1 + x_2 + \ldots + x_r = n$, then as above the required number is the coefficient of $x^n$ in

$$(x^0+x^1+\ldots+x^n)(x^0+x^1+\ldots+x^n)$$
$$\ldots(x^0+x^1+\ldots+x^n)(r\text{-brackets})$$

= Coefficient of $x^n$ in $(1 + x + x^2 + \ldots + x^n)^r$
= Coefficient of $x^n$ in $(1 + x + x^2 + \ldots)^r$
= Coefficient of $x^n$ in $(1 - x)^{-r}$
= Coefficient of $x^n$ in

$$\left[1+(-r)(-x)+\frac{(-r)(-r-1)}{2!}(-x)^2+\frac{(-r)(-r-1)(-r-2)}{3!}(-x)^3+\ldots\right]$$

= Coefficient of $x^n$ in

$$[1+{}^{r}C_1 x+{}^{r+1}C_2 x^2+{}^{r+2}C_3 x^3+\ldots]$$
$$={}^{n+r-1}C_n={}^{n+r-1}C_{r-1}$$

**Note :** If there are $l$ objects of one kind, $m$ objects of second kind, $n$ objects of third kind and so on; then the number of ways of choosing $r$ objects out of these objects is the coefficient of $x^r$ in the expansion of

$(1 + x + x^2 + x^3 + ... + x^l)\ (1 + x + x^2 + ... + x^m)\ (1 + x + x^2 + ... + x^n) \ldots$ .

Further if one object of each kind is to be included, then the number of ways of choosing $r$ objects out of these objects is the coefficient of $x^r$ in the expansion of

$(x + x^2 + x^3 + ... + x^l)\ (x + x^2 + x^3 + ... + x^m)\ (x + x^2 + x^3 + ... + x^n) \ldots$

and the number of possible permutations of $r$ objects out of these objects is the coefficient of $x^r$ in

$$r!\left(1+\frac{x}{1!}+\frac{x^2}{2!}+...+\frac{x^l}{l!}\right)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+...+\frac{x^m}{m!}\right)\left(1+\frac{x}{1!}+\frac{x^2}{2!}+...+\frac{x^n}{n!}\right)\ldots$$

While applying it, we use the coefficient of $x^r$ in $(1 - x)^{-n}$ is ${}^{n+r-1}C_r$ or ${}^{n+r-1}C_{n-1}$.

**(i)** Number of non-negative integral solution of equation $x_1+x_2+...+x_r=n$ is ${}^{n+r-1}C_n$ or ${}^{n+r-1}C_{r-1}$.

**(ii)** Number of positive integral solution of equation $x_1 + x_2 + ... + x_r = n$ is ${}^{n-1}C_{n-r}$ or ${}^{n-1}C_{r-1}$.

## PROBLEMS

## SECTION-I

### Single Correct Answer Type

**1.** Total number of integers '$n$' such that $2 \le n \le 2000$ and H.C.F of '$n$' and 36 is one, is equal to

(a) 666 (b) 667 (c) 665 (d) 668

**2.** A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time or any combination of 1s and 2s. The total number of ways in which the person can go up the stairs is

(a) 75 (b) 79 (c) 85 (d) 89

**3.** The number of ways of forming an arrangement of 5 letters from the letters of the word "IITJEE" is

(a) 60 (b) 96 (c) 120 (d) 180

**4.** Let $A = \{x_1, x_2, x_3, \ldots, x_7\}$, $B = \{y_1, y_2, y_3\}$. The total number of functions $f : A \to B$ that are onto and there are exactly three elements $x$ in $A$ such that $f(x) = y_2$, is equal to

(a) $14 \cdot {}^7C_2$ (b) $14 \cdot {}^7C_3$
(c) $7 \cdot {}^7C_2$ (d) $7 \cdot {}^7C_3$

**5.** The number of times the digit 3 will be written when listing the integers from 1 to 1000, is

(a) 269 (b) 300 (c) 271 (d) 302

**6.** The number of different ordered triplets $(a, b, c)$, $a, b, c \in I$ such that these can represent sides of a triangle whose perimeter is 21, is

(a) 12 (b) 31 (c) 55 (d) 91

**7.** The number of different permutations of all the letters of the word 'PERMUTATION' such that any two consecutive letters in the arrangement are neither both vowels nor both identical is

(a) $63 \times 6! \times 5!$ (b) $57 \times 5! \times 5!$
(c) $33 \times 6! \times 5!$ (d) $7 \times 7! \times 5!$

**8.** The least positive integral value of $x$ which satisfies the inequality ${}^{10}C_{x-1} > 2 \cdot {}^{10}C_x$ is

(a) 7 (b) 8 (c) 9 (d) 10

**9.** A man moves one unit distance for each step he takes. He always moves forward, backward, up or down either parallel to the $x$-axis or $y$-axis. He starts from the point (0, 0) and reaches the point (1, 1) at the end of six steps. The number of ways he can do it is

(a) 280 (b) 300 (c) 360 (d) 420

**10.** Number of pairs of positive integers $(p, q)$ whose L.C.M. (least common multiple) is 8100, is "$K$". Then number of ways of expressing $K$ as a product of two coprime numbers is

(a) 2 (b) 6 (c) 4 (d) 8

**11.** Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty ?

(a) 20 (b) 100 (c) 150 (d) 200

**12.** The number of points $(x, y, z)$ in space, whose each coordinate is a negative integer such that $x + y + z + 12 = 0$ , is

(a) 55 (b) 110 (c) 75 (d) 100

**13.** The number of positive terms in the sequence $x_n = \dfrac{195}{4^n P_n} - \dfrac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}, n \in N$ is

(a) 2 (b) 3 (c) 4 (d) 5

**14.** 12 small sticks of length 1 cm each are distributed into three children $A$, $B$ and $C$. These children join the sticks in the form of line segments individually. If

'$n$' is the number of ways in which the sticks can be distributed to the children so that the line segments joined by them form a triangle then the value of '$n$' is
(a) 10 (b) 9 (c) 8 (d) 12

**15.** A guard of 12 men is formed from a group of '$n$' soldiers. It is found that 2 particular soldiers $A$ and $B$ are 3 times as often together on guard as 3 particulars soldiers $C$, $D$ and $E$. Then $n - 24$ is equal to
(a) 8 (b) 7 (c) 2 (d) 6

**16.** Number of 4 digit numbers having the sum of the digits equal to 9 is ${}^{11}C_r$ then least value of $r$ =
(a) 1 (b) 2 (c) 3 (d) 4

**17.** Let $x_1x_2x_3x_4x_5x_4x_3x_2x_1$ be a nine digit palindrome such that either the sequence $(x_1, x_2, x_3, x_4, x_5)$ is a strictly ascending or strictly descending. Then the number of such palindromes is
(a) $9 \times {}^9P_4$ (b) $3 \times {}^9P_5$
(c) $9 \times {}^9C_5$ (d) $3 \times {}^9C_5$

**18.** The number of positive integers $\leq 100000$ which contain exactly one 2, one 5 and one 7 in its decimal representation is
(a) 2940 (b) 7350 (c) 2157 (d) 1582

**19.** If $x, y \in (0, 30)$ such that

$$\left[\frac{x}{3}\right]+\left[\frac{3x}{2}\right]+\left[\frac{y}{2}\right]+\left[\frac{3y}{4}\right]=\frac{11}{6}x+\frac{5}{4}y$$

(where $[x]$ denotes greatest integer $\leq x$). Then the number of ordered pairs $(x,y)$ is
(a) 0 (b) 2 (c) 4 (d) 28

**20.** If $\sum_{r=0}^{n-1}\left(\frac{n_{C_r}}{n_{C_r}+n_{C_{r+1}}}\right)^3=\frac{4}{5}$, then $n$ =
(a) 5 (b) 4 (c) 3 (d) 6

**21.** $A$ (1, 2) and $B$(5, 5) are two points. Starting from $A$, line segments of unit length are drawn either rightwards or upwards only, in each step, until $B$ is reached. Then, the number of ways of connecting $A$ and $B$ in this manner is
(a) 35 (b) 40 (c) 45 (d) 50

**22.** Let the product of all the divisors of 1440 be $P$. If $P$ is divisible by $24^x$, then the maximum value of $x$ is
(a) 28 (b) 30 (c) 32 (d) 36

**23.** The number of 5-digit numbers which are divisible by 3 that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9, when repetition of digits is allowed, is
(a) $3^9$ (b) $4{\cdot}3^8$ (c) $5{\cdot}3^8$ (d) $7{\cdot}3^8$

**24.** The number of 4-digit numbers that can be formed by using the digits 1,2,3,4,5,6,7,8 and 9 such that the least digit used is 4, when repetition of digits is allowed, is
(a) 617 (b) 671 (c) 716 (d) 761

## SECTION-II

### Multiple Correct Answer Type

**25.** ${}^{11}C_{10} \cdot {}^9C_1 + {}^{11}C_9 \cdot {}^9C_2 + \ldots\ldots\ldots + {}^{11}C_2 \cdot {}^9C_9 =$
(a) ${}^{20}C_{11}$ (b) ${}^{20}C_8$ (c) ${}^{20}C_9 - 1$
(d) Number of different ways of exchanging 11 books of $A$ with the 9 books of $B$

**26.** Using the elements $-3, -2, -1, 0, 1, 2, 3$
(a) The number of $3 \times 3$ matrices having trace 0 is $37(7^6)$
(b) The number of $3 \times 3$ matrices is $7^9$
(c) The number of $3 \times 3$ skew symmetric matrices is $7^3$
(d) The number of $3\times 3$ symmetric matrices is $7^6$

**27.** Consider the following statements.
(i) In a 12 storeyed house, 10 people enter the lift cabin at ground floor. It is known that they will leave lift in groups of particular 2, 3 and 5 people at different storey. The number of ways this can be done if the lift does not stop at first and second floors is 720.
(ii) Each of three ladies have brought their one child for admission to a school. The principal wants to interview the six persons one by one, subject to the condition that no mother is interviewed before her child. The number of ways in which interviews can be arranged is 90.
(iii) The number of ways in which one can put three balls numbered 1, 2, 3 in three boxes labelled $a$, $b$, $c$ such that atmost one box is empty is equal to 18.
(iv) A box contains 5 different red balls and 6 different white balls. The total number of ways in which 4 balls can be selected, taking atleast 1 ball of each colour is 310.
(a) statements (i) and (ii) are correct.
(b) statements (ii) and (iv) are correct.
(c) statements (i) and (iii) are correct.
(d) all statements are correct.

**28.** A fair coin is tossed $n$ times. Let $a_n$ denotes the no. of cases in which no two heads occur consecutively, then
(a) $a_1 = 2$ (b) $a_2 = 3$
(c) $a_5 = 14$ (d) $a_8 = 55$

**29.** If $X = 144$, then

(a) no. of divisors (including 1 and $X$) of $X = 15$

(b) sum of divisors (including 1 and $X$) of $X = 403$

(c) product of divisors (including 1 and $X$) of $X = 12^{15}$

(d) sum of reciprocals of divisors (including 1 and $X$) of $X = \frac{403}{144}$

**30.** Letters of the word SUDESH can be arranged in

(a) 120 ways when two vowels are together

(b) 180 ways when two vowels occupy in alphabetical order

(c) 24 ways when vowels and consonants occupy their respective places

(d) 240 ways when vowels do not occur together

**31.** The number of isosceles triangles with integer sides if no side exceeds 2008 is

(a) $(1004)^2$ if equal sides do not exceed 1004.

(b) $2(1004)^2$ if equal sides exceed 1004.

(c) $3(1004)^2$ if equal sides have any length $\le 2008$.

(d) $(2008)^2$ if equal sides have any length $\le 2008$.

**32.** A contest consists of ranking 10 songs of which 6 are Indian classic and 4 are western songs. Number of ways of ranking so that

(a) There are exactly 3 Indian classic songs in top 5 is $(5!)^3$.

(b) Top rank goes to Indian classic song is 6(9!).

(c) The ranks of all western songs are consecutive is 4! 7!.

(d) The 6 Indian classic songs are in a specified order is ${}^{10}P_4$.

**33.** Let for $n \in N$, $f(n) = \sum_{r=0}^{n} (-1)^r \frac{{}^nC_r 2^{r+1}}{(r+1)(r+2)}$ then

(a) $f(2n) = f(2n+1)$ (b) $f(n) = f(n+1)$

(c) $f(2n) = f(2n-1)$ (d) $f(2011) = f(2012)$

**34.** Which of the following is/are true?

(a) $5^6 - {}^5C_1 \cdot 4^6 + {}^5C_2 \cdot 3^6 - {}^5C_3 \cdot 2^6 + {}^5C_4 \cdot 1^6 = {}^6C_2 \cdot 5!$

(b) $5^6 - {}^6C_1 \cdot 5^5 + {}^6C_2 \cdot 4^5 - {}^6C_3 \cdot 3^5 + {}^6C_4 \cdot 2^5 - {}^6C_1 \cdot 1^5 = 0$

(c) $6^6 - {}^6C_1 \cdot 5^6 + {}^6C_2 \cdot 4^6 - {}^6C_3 \cdot 3^6 + {}^6C_4 \cdot 2^6 - {}^6C_5 \cdot 1^6 = 720$

(d) $6^5 - {}^6C_1 \cdot 5^5 + {}^6C_2 \cdot 4^5 - {}^6C_3 \cdot 3^5 + {}^6C_4 \cdot 2^5 - {}^6C_5 \cdot 1^5 = {}^5C_2 \cdot 6!$

## SECTION-III

### Comprehension Type

**Paragraph for Question No. 35 to 37**

10-digit numbers are formed by using all the digits 0,1,2,3,4,5,6,7,8 and 9 such that they are divisible by 11111.

**35.** The digit in the ten's place, in the smallest of such numbers, is

(a) 9 (b) 8 (c) 6 (d) 7

**36.** The digit in the unit's place, in the greatest of such numbers, is

(a) 2 (b) 3 (c) 4 (d) 1

**37.** The total number of such numbers is

(a) 3456 (b) 5634 (c) 6543 (d) 4365

**Paragraph for Question No. 38 to 40**

Define a function $\phi : N \to N$ as follows : $\phi(1) = 1, \phi(P^n) = P^{n-1}(P-1)$ if $P$ is prime and $n \in N$ and $\phi(mn) = \phi(m)\phi(n)$ if $m$ & $n$ are relatively prime natural numbers.

**38.** $\phi(8n+4)$ where $n \in N$ is equal to

(a) $\phi(4n+2)$ (b) $\phi(2n+1)$

(c) $2\phi(2n+1)$ (d) $4\phi(2n+1)$

**39.** The number of natural numbers '$n$' such that $\phi(n)$ is odd is

(a) 1 (b) 2 (c) 3 (d) 4

**40.** If $\phi(7^n) = 2058$ where $n \in N$, then the value of $n$ is

(a) 3 (b) 4 (c) 5 (d) 6

**Paragraph for Question No. 41 to 43**

If a set $A$ has $n$ elements then the number of subsets of $A$ containing exactly $r$ elements is ${}^nC_r$. The number of all subsets of $A$ is $2^n$. Now answer the following questions. A set $A$ has 7 elements. A subset $P$ of $A$ is selected. After noting the elements they are placed back in $A$. Again subset $Q$ is selected. Then the number of ways of selecting $P$ and $Q$ such that

**41.** $P$, $Q$ have no common element is

(a) 2830 (b) 124 (c) 3432 (d) 2187

**42.** $P$ and $Q$ have exactly 3 elements in common is

(a) 2835 (b) 125 (c) 3430 (d) 2185

**43.** $P$ and $Q$ have equal number of elements is ($P$ and $Q$ may be null sets)

(a) 2834 (b) 120 (c) 3432 (d) 2180

**Paragraph for Question No. 44 to 46**

Two numbers $x$ and $y$ are drawn without replacement from the set of the first 15 natural numbers. The number of ways of drawing them such that

**44.** $x^3 + y^3$ is divisible by 3

(a) 24 (b) 32 (c) 35 (d) 65

**45.** $x^2 - y^2$ is divisible by 5

(a) 21 (b) 33 (c) 30 (d) 69

**46.** $x^4 - y^4$ is divisible by 5

(a) 57 (b) 64 (c) 69 (d) 72

## SECTION-IV

### Matrix-Match Type

**47.** Match the following:

There are 2 Indian couples, 2 American couples and one unmarried person

| Column I | | Column II | |
|---|---|---|---|
| (A) | The total number of ways in which they can sit in a row such that an Indian wife and an American wife are always on either side of the unmarried person, is | (p) | 22680 |
| (B) | The total number of ways in which they can sit in a row such that the unmarried person always occupy the middle position, is | (q) | 5760 |
| (C) | The total number of ways in which they can sit around a circular table such that an Indian wife and an American wife are always on either side of the unmarried person, is | (r) | 40320 |
| (D) | If all the nine persons are to be interviewed one by one then the total number of ways of arranging their interviews such that no wife gives interview before her husband, is | (s) | 24320 |

**48.** Match the following:

Consider all possible permutations of the letters of the word MASTERBLASTERS.

| Column I | | Column II | |
|---|---|---|---|
| (A) | The number of permutations containing the word RAAT is | (p) | $\frac{(7!)^2}{3!(2!)^4}$ |
| (B) | The number of permutations in which S occurs in first place and R occurs in the last place is | (q) | $\frac{11!\times 4!}{3!\times(2!)^2}$ |
| (C) | The number of permutations in which none of the letters S, T, R occur in first 7 positions is | (r) | $\frac{11!}{3!\times 2!}$ |
| (D) | The number of permutations in which the letters A, S, R occur in even positions is | (s) | $\frac{12!}{(2!)^4}$ |

**49.** Match the following

| Column I | | Column II | |
|---|---|---|---|
| (A) | Number of ways of selecting 8 balls out of an unlimited collection of red, blue, green and yellow balls is | (p) | $^{21}C_3$ |
| (B) | Number of 4 digit numbers having the sum of the digits equal to 9 is | (q) | Number of ways of distributing 12 apples to four people with each one getting atleast one |
| (C) | Number of ways of arranging 3 identical red balls, 20 identical white balls in a row so that no two red balls are together is | (r) | 6 |
| (D) | Exponent of 3 in 17! is | (s) | $^{11}C_3$ |

## SECTION-V

### Integer Answer Type

**50.** If $n > 1$ is the smallest integer with the property that $n^2(n-1)$ is divisible by 2009, then the integral part of $\frac{n}{8}$ is

**51.** The number of onto functions which are non-decreasing from $A = \{1, 2, 3, 4, 5\}$ to $B = \{7, 8, 9\}$ is

**52.** Let $S = \{1, 2, 3, \ldots, n\}$, If $X$ denote the set of all subsets of $S$ containing exactly two elements, then the value of $\sum_{A \in X} (\min A)$ is $^{n+1}C_\lambda$ then $\lambda$ =

**53.** If one test (on screening paper basis) was conducted on Batch $A$, maximum number of marks is $(90 \times 3) = 270$. 4 students get the marks lower than 80.

Coaching institute decide to inform their guardians, that is why their result card were sent to their home. The number of ways, in which all the letters were put in wrong envelopes, is

**54.** The exponent of 7 in $^{100}C_{50}$ is

**55.** The unit digit in 1! + 2! + 3! + ….. + 49! is

**56.** Nine hundred distinct $n$-digit numbers are to be formed using exactly the three digits 2, 5 and 7. The smallest value of $n$ for which this is possible, is

**57.** The number of different ordered triplets $(a, b, c)$, $a, b, c \in I$ such that these can represent sides of a triangle whose perimeter is 21, is $9k+10$,then $k$ is

**58.** Out of 5 apples, 10 mangoes and 15 oranges, the number of ways of distributing 15 fruits each to two persons, is $n$, then $\frac{n}{22}$ is equal to

**59.** The number of divisors of the form $2n + 1$ $(n \geq 1)$ of the integer 120 is

## SOLUTIONS

**1. (a) :** $36 = 2^2 \times 3^2$

From 2 to 2000, number of multiples of 2 are $\left[\frac{2000}{2}\right] = 1000$

From 2 to 2000, number of multiples of 3 are $\left[\frac{2000}{3}\right] = 666$

From 2 to 2000, number of multiples of 6 are $\left[\frac{2000}{6}\right] = 333$

Number of possible '$n$' are

$= 1999 - [1000 + 666 - 333] = 666$

**2. (d) :** A flight of stairs has 10 steps. A person can go up the steps one at a time, two at a time or any combination of 1s and 2s.

Let $x + 2y = 10$

where $x$ is the number of times he takes single steps and $y$ is the number of times he takes two steps.

| **Cases** | **Total no. of ways** |
|---|---|
| I : When $x = 0$ and $y = 5$ | $\frac{5!}{5!} = 1 (2\,2\,2\,2\,2)$ |
| II : When $x = 2$ and $y = 4$ | $\frac{6!}{2!\,4!} = 15 (1\,1\,2\,2\,2\,2)$ |
| III : When $x = 4$ and $y = 3$ | $\frac{7!}{4!3!} = 35 (1\,1\,1\,1\,2\,2\,2)$ |
| IV : When $x = 6$ and $y = 2$ | $\frac{8!}{2!6!} = 28 (1\,1\,1\,1\,1\,1\,2\,2)$ |
| V : When $x = 8$ and $y = 1$ | $^9C_1 = 9 (1\,1\,1\,1\,1\,1\,1\,1\,2)$ |
| VI : When $x = 10$ and $y = 0$ | $1 (1\,1\,1\,1\,1\,1\,1\,1\,1\,1)$ |

Hence, total no. of ways

$= 1 + 15 + 35 + 28 + 9 + 1 = 89.$

**3. (d) :** Number of arrangements in which two are identical of one kind, two are identical of another kind and one letter different from the remaining two letters is $^2C_1 \times \frac{5!}{(2!)^2} = 60.$

Number of arrangements in which 2 are identical of one kind and the rest are different is $^2C_1 \times \frac{5!}{2!} = 120$

**4. (b) :** $A = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$,

$B = \{y_1, y_2, y_3\}$

$f : A \to B$ is onto such that $f(x) = y_2$

Since exactly three elements of $A$ has image $y_2$

$\therefore$ it can be done in $^7C_3$ ways.

Remaining four elements of $A$ is associated with 2 elements of $B$ in $2^4 - {}^2C_1(2 - 1)^4 = 14$ ways

Total no. of onto functions = $^7C_3 \times 14$

**5. (b) :** Since 3 does not occur in 1000, we have to count the number of times 3 occurs when we list the integers from 1 to 999. Any number between 1 and 999 is of the form $xyz$, where $0 \leq x, y, z \leq 9$. Let us first count the number in which 3 occurs exactly once. Since 3 can occur at one place in $^3C_1$ ways, there are $^3C_1\,(9 \times 9) = 3 \times 9^2$ such numbers. Next, 3 can occur in exactly two places in $(^3C_2)(9) = 3 \times 9$ such numbers. Lastly, 3 can occur in all three digits in one number only. Hence, the number of times 3 occurs is

$1 \times (3 \times 9^2) + 2 \times (3 \times 9) + (3 \times 1) = 300.$

**6. (c) :** $a + b + c = 21 \Rightarrow b + c > a$

$\Rightarrow \; a + b + c > 2a \Rightarrow 2a < 21 \Rightarrow a \leq 10.$

So, $1 \leq a, b, c \leq 10$.

The cases when $a > b > c$ are (10, 9, 2), (10, 8, 3), (10, 7, 4), (10, 6, 5), (9, 8, 4), (9, 7, 5) and (8, 7, 6). So, number of cases when $a, b, c$ are all distinct is $7 \times 3! = 42$.

The cases when $a = b > c$ or $a > b = c$ are (10, 10, 1), (9, 9, 3), (8, 8, 5) and (9, 6, 6). So, number of cases when two same and 1 different is $4 \times (3!\,/2!) = 12$.

The cases when $a = b = c$ is (7, 7, 7). The total number of ordered triplets = 42+12 + 1 = 55.

**7. (b) :** The letters other than vowels are : PRMTTN

Number of permutations with no two vowels together is $\frac{6!}{2!} \times {}^7C_5 \times 5!$

Further among these permutations, the number of cases in which T's are together is $5! \times {}^6C_5 \times 5!$

So, the required number

$$= \frac{6!}{2!}\,{}^{7}C_5 \times 5! - 5! \times {}^{6}C_5 \times 5! = 57 \times (5!)^2$$

**8. (b) :** $10 \geq x - 1 \Rightarrow x \leq 11$ and $10 \geq x$

$\therefore \quad x \leq 10$

$$\because \quad {}^{10}C_{x-1} > 2 \cdot {}^{10}C_x \Rightarrow 1 > 2 \cdot \frac{{}^{10}C_x}{{}^{10}C_{x-1}}$$

$$\Rightarrow \quad 1 > 2 \cdot \frac{10 - x + 1}{x} \quad \Rightarrow \quad x > 22 - 2x \Rightarrow x > \frac{22}{3}$$

$$\Rightarrow \quad x > 7\frac{1}{3} \quad \therefore \; x = 8$$

**9. (b) :** Let E, W, N, S stand for one unit movement along +ve, −ve $x$-direction, +ve, −ve $y$-direction respectively. The sequence of 6 steps are

ENNNSS with $\frac{6!}{3!2!} = 60$ ways

EEWNNS with $\frac{6!}{2!2!} = 180$ ways

EEEWWN with $\frac{6!}{3!2!} = 60$ ways

The desired number = 60 + 180 + 60 = 300

**10. (a) :** L.C.M. $(p,q) = 2^2 3^4 5^2$

$p = 2^{a_1} 3^{b_1} 5^{c_1}, q = 2^{a_2} 3^{b_2} 5^{c_2}$

$\Rightarrow$ max. $\{a_1, a_2\} = 2 \Rightarrow 5$ ways

$\Rightarrow$ max. $\{b_1, b_2\} = 4 \Rightarrow 9$ ways

$\Rightarrow$ max. $\{c_1, c_2\} = 2 \Rightarrow 5$ ways

$\therefore$ $K = 3^2 \cdot 5^2$ can be expressed as $1 \cdot 3^2 5^2$, $3^2 \cdot 5^2$

**11. (c) :** $\overline{3}\,\overline{1}\,\overline{1}$ $\quad {}^{5}C_3 \cdot {}^{2}C_1 \cdot {}^{1}C_1 \left(\frac{3!}{2!}\right)$

$2\,2\,1$ $\quad {}^{5}C_2 \cdot {}^{3}C_2 \cdot {}^{1}C_1 \left(\frac{3!}{2!}\right)$

**12. (a) :** $x + y + z + 12 = 0$, $x, y, z$ are negative integers. Let $x = -a$, $y = -b$, $z = -c$, $a, b, c$ are positive integers.

Then, required number of points $(x, y, z)$

= number of positive integral solutions of $a + b + c = 12$

= coefficient of $x^{12}$ in $(x + x^2 + \ldots)^3$

= coefficient of $x^9$ in $(1 - x)^{-3} = {}^{11}C_2 = \frac{11 \times 10}{2} = 55$

**13. (c) :** We have, $x_n = \frac{195}{4\,{}^{n}P_n} - \frac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}, n \in N$

$$= \frac{195}{4 \cdot n!} - \frac{(n+3)(n+2)(n+1)}{(n+1)!} = \frac{171 - 4n^2 - 20n}{4 \cdot n!}$$

$\because$ $x_n$ is positive.

$$\therefore \quad \frac{171 - 4n^2 - 20n}{4 \cdot n!} > 0 \Rightarrow 4n^2 + 20n - 171 < 0$$

which is true for $n$ = 1, 2, 3, 4.

Hence, the given sequence has 4 positive terms.

**14. (a) :** Let $x, y, z$ be the number of sticks received by the children $A$, $B$ and $C$. Then the line segments formed by them form a triangle iff $1 \leq x \leq 5$, $1 \leq y \leq 5$, $1 \leq z \leq 5$

$\therefore$ $n$ is the number of positive integral solutions of the equation $x + y + z = 12$ where $x, y, z \leq 5$.

$\Rightarrow$ $n$ = coefficient of $x^{12}$ in $(x + \ldots + x^5)^3$

= coefficient of $x^9$ in $(1 + x + x^2 + x^3 + x^4)^3$

= coefficient of $x^9$ in $(1 - x^5)^3 (1 - x)^{-3}$

= coefficient of $x^9$ in $(1 - 3x^5 + 3x^{10} - x^{15})\,(1 - x)^{-3}$

$= 1 \times {}^{11}C_2 - 3 \times {}^{6}C_2 = 55 - 45 = 10$

**15. (a) :** ${}^{n-2}C_{12-2} = 3 \cdot {}^{n-3}C_{12-3}$

$\Rightarrow$ ${}^{n-2}C_{10} = 3 \cdot {}^{n-3}C_9$

$\Rightarrow$ $\frac{n-2}{10} = 3 \Rightarrow n = 32$

**16. (c) :** $x_1 + x_2 + x_3 + x_4 = 9$, $x_1 \neq 0$

Coefficient of $x^9$ in $(x + x^2 + \ldots + x^9)(1 + x + \ldots + x^8)^3$

Coefficient of $x^8$ in $(1 + x + \ldots + x^8)^4 = {}^{8+4-1}C_8 = {}^{11}C_3$

**17. (d) :** Strictly descending $\rightarrow {}^{10}C_5$

Strictly ascending $\rightarrow {}^{9}C_5$ (because zero can't be at $x_1$)

${}^{10}C_5 + {}^{9}C_5 = 2 \cdot {}^{9}C_4 + {}^{9}C_4 = 3 \cdot {}^{9}C_4 = 3 \cdot {}^{9}C_5$

**18. (a) :** Take 5 gaps _ _ _ _ _

The digit '2' can occupy any of 5 places

The digit '5' can occupy any of 4 places

The digit '7' can occupy any of 3 places

Remaining 2 places in $7 \times 7$ ways

$\therefore$ Total ways $= 5 \times 4 \times 3 \times 7 \times 7 = 2940$

**19. (d) :** From the given condition,

$$\left\{\frac{x}{3}\right\} + \left\{\frac{3x}{2}\right\} + \left\{\frac{y}{2}\right\} + \left\{\frac{3y}{4}\right\} = 0$$

$\Rightarrow$ $\frac{x}{3}, \frac{3x}{2}, \frac{y}{2}$ and $\frac{3y}{4}$ must be integers.

$\therefore$ $x = 6, 12, 18, 24$

$y = 4, 8, 12, 16, 20, 24, 28$

$\therefore$ No. of ordered pairs $= 4 \times 7 = 28$

**20. (b) :** L.H.S.

$$= \sum_{r=0}^{n-1} \left( \frac{{}^{n}C_r}{{}^{n}C_r + {}^{n}C_{r+1}} \right)^3 = \sum_{r=0}^{n-1} \left( \frac{{}^{n}C_r}{{}^{n}C_r \left(1 + \frac{{}^{n}C_{r+1}}{{}^{n}C_r}\right)} \right)^3$$

$$= \sum_{r=0}^{n-1}\left(\frac{r+1}{n+1}\right)^3 = \frac{n^2(n+1)^2}{4(n+1)^3} = \frac{n^2}{4(n+1)}$$

Now $\frac{n^2}{4(n+1)} = \frac{4}{5} \Rightarrow 5n^2 - 16n - 16 = 0$

$n = 4$ or $n = -\frac{4}{5} \Rightarrow n = 4$

**21. (a) :** Given $A(1,2)$ and $B(5,5)$.
Difference of $x$-coordinates = 5 – 1 = 4
∴ Exactly 4 rightward steps are needed.
Difference of $y$-coordinates = 5 – 2 = 3.
∴ Exactly 3 upward steps are needed.
**Note:** Order of the steps is immaterial.
Denote each rightward step by R and each upward step by U.
The problem is arranging the letters RRRRUUU

No. of arrangements $= \frac{7!}{4!3!} = 35$

**22. (b) :** $1440 = 2^5 \cdot 3^2 \cdot 5^1$
No. of divisors = (5 + 1)·(2 + 1)·(1 + 1) = 36
Product of divisors = 1·2·3……. 480·720·1440. Here all the 36 divisors are written in the increasing order. They can be clubbed into 18 pairs as shown below.
(1,1440), (2, 720), (3, 480) …… etc.
∴ Product of divisors = $(1440)^{18} = 2^{90} \cdot 3^{36} \cdot 5^{18}$
$= (2^3 \cdot 3)^{30} \cdot 3^6 \cdot 5^{18} = 24^{30} \cdot 3^6 \cdot 5^{18}$
which is divisible by $24^x$
∴ Maximum value of $x$ = 30

**23. (a) :** _ _ _ _ _ (5 blanks)
1st blank can be filled in 9 ways
2nd blank can be filled in 9 ways (since repetition is allowed).
3rd blank can be filled in 9 ways
4th blank can be filled in 9 ways
Now, we have to fill the 5th blank carefully such that the number is divisible by 3. Add the 4 numbers in the first 4 blanks.
If their sum is in the form $3n$, then fill the last blank by 3, 6 or 9 so that the sum of all digits is divisible by 3.
If their sum is in the form $3n+1$, then fill the last blank by 2, 5 or 8.
If their sum is in the form $3n+2$, then fill the last blank by 1, 4 or 7.
Therefore, in any case, the last blank can be filled in 3 ways only.
∴ Ans = 9 × 9 × 9 × 9 × 3 = $3^9$

**24. (b) :** Least digit used = 4
∴ We can use 4,5,6,7,8,9. But remember that atleast one 4 must be used _ _ _ _ (4 blanks)
1st blank can be filled in 6 ways.
2nd blank can be filled in 6 ways.
3rd blank can be filled in 6 ways.
4th blank can be filled in 6 ways.
∴ 4 blanks can be filled in $6^4$ ways. But out of these, some may contain no 4 at all. Let us find them.
_ _ _ _ (4 blanks)
Each blank can be filled in 5 ways (by 5, 6, 7, 8, or 9)
∴ $5^4$ ways (no 4 at all)
∴ Atleast one 4 = $6^4 - 5^4 = 671$.

**25. (c, d) :** $(1+x)^{11} = {}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + \ldots + {}^{11}C_{11}x^{11}$ … (i)
$(1+x)^9 = {}^9C_0 + {}^9C_1x + {}^9C_2x^2 + \ldots + {}^9C_9x^9$ … (ii)
On multiply (i) & (ii) and compare coefficient of $x^{11}$ on both sides and put $x$ = 1
${}^{20}C_{11} = {}^{11}C_{11}{}^9C_0 + {}^{11}C_{10}{}^9C_1 + \ldots + {}^{11}C_2{}^9C_9$
∴ ${}^{20}C_9 - 1 = {}^{11}C_{10}{}^9C_1 + \ldots + {}^{11}C_2{}^9C_9$

**26. (a, b, c, d) :** (a) $a_{11} + a_{22} + a_{33} = 0$, remaining '6' elements can be filled in $7^6$ ways.

$\left.\begin{matrix}(-3,0,3),(-2,0,2),(-1,0,1)\\(-3,1,2),(3,-1,-2)\end{matrix}\right\} \rightarrow 3! \cdot 5 = 30$

(–2, 1, 1), (2, –1, –1) → 3 · 2 = 6
(0, 0, 0) → 1
(b) Each of 9 elements can be filled in 7 ways.
(c) $\begin{bmatrix} 0 & - & - \\ & 0 & - \\ & & 0 \end{bmatrix}$, 3 elements can be filled in $7^3$ ways.
(d) $a_{11}$, $a_{22}$, $a_{33}$ filled in 7 ways, also $a_{12}$ filled in 7 ways then $a_{21}$ filled in one way and so on
$\begin{bmatrix} - & - & - \\ & - & - \\ & & - \end{bmatrix}$, 6 elements can be filled in $7^6$ ways.

**27. (a, b) :** (i) The number of ways = ${}^{10}C_3 \times 3!$
= 720
(ii) Each lady and her child can be arranged in a fixed order only.
∴ The total number of ways in which interview can be held $= \frac{6!}{2!2!2!} = 90$
(iii) **Case I:** No box empty
Then the number of ways = 3! = 6

**Case II:** If one of the boxes is empty, then number of ways = ${}^3C_1(2^3 - 2) = 18$.
$\therefore$ total number of ways = 6 + 18= 24
(iv) Total – (All red) – (All white)
$= {}^{11}C_4 - {}^5C_4 - {}^6C_4 = 330 - 5 - 15 = 310$

**28. (a, b, d) :** The cases for $a_1$ : {H, T} *i.e.*, $a_1 = 2$
The case for $a_2$ : {HT, TH, TT}, $a_2 = 3$
For $n \geq 3$, If the first outcome is H then next just T and then $a_{n-2}$. If the first outcome is $T$ then $a_{n-1}$ should follow.
So, $a_n = 1 \times 1 \times a_{n-2} + 1 \times a_{n-1} \Rightarrow a_n = a_{n-2} + a_{n-1}$
So, $a_3 = a_1 + a_2 = 5$, $a_4 = 3 + 5 = 8$ and so on.

**29. (a, b, c, d) :** $144 = 2^4 \cdot 3^2$
(a) No. of divisors = $(4 + 1)\cdot(2 + 1) = 15$
(b) Sum of divisors = $(1+2+2^2+2^3+2^4)\ (1+3+3^2)$
$= 403$
(c) Product of divisors $= (144)^{\frac{15}{2}} = (12)^{15}$
(d) Sum of reciprocals of divisors
$$= \frac{\text{sum of divisors}}{144} = \frac{403}{144}$$

**30. (a, b, c, d) :** (a) $(2!)\frac{5!}{2!}$ (b) $\frac{6!}{2!\,2!}$
(c) $\frac{4!}{2!}(2!)$ (d) 360 – 120

**31. (a, b, c) :** If the sides are $a$, $a$, $b$, then the triangle forms only when $2a > b$, so for any $a \in N$, $b$ can change from 1 to $2a - 1$. When $a \leq 1004$ then number of triangles = $1 + 3 + 5 + \ldots + (2(1004) - 1) = (1004)^2$ and if $1005 \leq a \leq 2008$, $b$ can take any value from 1 to 2008.
But $a$ has 1004 possibilities, hence number of triangles $= 1004 \times 2008 = 2(1004)^2$
$\therefore$ Total number of isosceles triangles = $3(1004)^2$

**32. (a, b, c, d) :** (a) ${}^6C_3 \cdot {}^4C_2 \cdot 5! \cdot 5! = (5!)^3$
(b) ${}^6C_1 \cdot 9!$ (c) $(6 + 1)!\ 4!$ (d) ${}^{10}P_4$

**33. (c, d) :** $$f(n) = \frac{1}{2(n+1)(n+2)} \sum_{r=0}^{n} {}^{n+2}C_{r+2}(-2)^{r+2}$$
$$= \frac{(1-2)^{n+2} - 1 + 2(n+2)}{2(n+1)(n+2)} = \begin{cases} \frac{1}{n+2}, \text{ if } n = \text{odd} \\ \frac{1}{n+1}, \text{ if } n = \text{even} \end{cases}$$

**34. (a, c) :** (a) Number of onto functions from a set containing 6 elements to a set containing 5 elements $= {}^6C_2 \cdot \lfloor 5$
(c) Number of onto functions from a set containing 6 elements to a set containing 6 elements $= \lfloor 6 = 720$

**35 - 37 :**
Let $a\ b\ c\ d\ e\ f\ g\ h\ i\ j$ be one of such numbers where $a\ b\ c\ d\ e\ f\ g\ h\ i\ j$ is some permutation of the digits 0,1,2,3,4,5,6,7,8,9 where $a \neq 0$.
Sum of digits of the number = 0+1+2+3+4+5+6+7+8+9= 45, which is divisible by 9 and hence the number is divisible by 9. But it is divisible by 11,111 also and 11,111 is not divisible by 3 or 9. Therefore, the number is divisible by11,111 × 9 = 99,999.
And $a\ b\ c\ d\ e\ f\ g\ h\ i\ j = a\ b\ c\ d\ e \times 10^5 + f\ g\ h\ i\ j$
$= a\ b\ c\ d\ e \times (99{,}999+1) + f\ g\ h\ i\ j$
$= a\ b\ c\ d\ e \times 99{,}999 + a\ b\ c\ d\ e + f\ g\ h\ i\ j$ is divisible by 99,999.
$\Rightarrow a\ b\ c\ d\ e + f\ g\ h\ i\ j$ is divisible by 99,999.
But $a\ b\ c\ d\ e < 99{,}999$
And $f\ g\ h\ i\ j < 99{,}999$
$\Rightarrow a\ b\ c\ d\ e + f\ g\ h\ i\ j < 2 \times 99{,}999$
$\therefore\ a\ b\ c\ d\ e + f\ g\ h\ i\ j = 99{,}999$
$\Rightarrow\ e + j = d + i = c + h = b + g = a + f = 9$

**35. (c) :**

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 2 | 3 | 4 | 8 | 9 | 7 | 6 | 5 |

For smallest number, $a$ must be 1 (since $a$ cannot be 0) and hence $f = 8$.
Then, $b = 0 \Rightarrow g = 9$
$c = 2 \Rightarrow h = 7$
$d = 3 \Rightarrow i = 6$
$e = 4 \Rightarrow j = 5$
$\therefore$ The smallest of such numbers is 1023489765 and the digit in the ten's place is 6.

**36. (c) :**

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ |
|---|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 0 | 1 | 2 | 3 | 4 |

For greatest number, $a = 9 \Rightarrow f = 0$
Then, $b = 8 \Rightarrow g = 1$
$c = 7 \Rightarrow h = 2$
$d = 6 \Rightarrow i = 3$
$e = 5 \Rightarrow j = 4$
$\therefore$ The greatest of such numbers is 9876501234 and the digit in the unit place is 4.

**37. (a) :**

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ |
|---|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 6 | 4 | 2 | 1 | 1 | 1 | 1 | 1 |

The blank '$a$' can be filled in 9 ways (except 0).
Then blank '$f$' can be filled in only one way (by 9–$a$).
Now, blank '$b$' can be filled by any of the remaining 8 digits.

Then blank '*g*' can be filled in only one way (by 9– *b*)
Now, blank '*c*' can be filled by any of the remaining 6 digits.
Then blank '*h*' can be filled in only one way (by 9 – *c*).
Now, blank '*d*' can be filled by any of the remaining 4 digits.
Then blank '*i*' can be filled in only one way (by 9 – *d*).
Now, blank '*e*' can be filled by any of the remaining 2 digits.
Then blank '*j*' can be filled in only one way (by 9 – *e*).
$\therefore$ The total number of such numbers
$= 9 \times 1 \times 8 \times 1 \times 6 \times 1 \times 4 \times 1 \times 2 \times 1 = 3456.$

**38 - 40 :**

**38. (c) :** $\phi(1)=1, \phi(P^n)=P^{n-1}(P-1), \phi(mn)=\phi(m)\cdot\phi(n)$

$\phi(8n+4)=\phi(4(2n+1))=\phi(4)\cdot\phi(2n+1)$

$=\phi(2^2)\cdot\phi(2n+1)=2\cdot\phi(2n+1)$

**39. (b) :** $\phi(n)$ is odd.
$\Rightarrow$ $\phi(P^n)$ is odd.
$\Rightarrow$ $P^{n-1}(P-1)$ is odd.
$\because$ $P$ is prime. The only value $P$ can take is $P = 2$
$\therefore$ $\phi(2^n)$is odd.
$\Rightarrow$ $2^{n-1}(2-1) = 2^{n-1}$ is odd.
$\Rightarrow$ $n - 1 = 0 \Rightarrow n = 1 \therefore \phi(1) = 1 = \phi(2)$

**40. (b) :** $\phi(7^n) = 2058$
$7^{n-1}(7-1) = 2058$
$7^{n-1} = 343$
$n - 1 = 3 \Rightarrow n = 4$

**41- 43 :**

**41. (d) :** If $P$ and $Q$ have no common element, then each of the 7 elements have 3 choices. They are
$x \in P$ and $x \notin Q$ or $x \notin P$ and $x \in Q$ or $x \notin P$ and $x \notin Q$
Hence, the number of ways of selecting $P$ and $Q$
$= 3^7 = 2187$

**42. (a) :** As in both sets three elements are common so, three elements can be choosen in ${}^7C_3$ ways and rest of the elements can be chosen in any of the above three ways in $3^4$ ways. So, total number of ways $={}^7C_3 \times 3^4 = 2835$

**43. (c) :** Let $P$ & $Q$ have same number of elements *i.e* $r\,(0 \le r \le 7)$
$\Rightarrow$ Total number of ways in which $P$ & $Q$ have same number of elements

$$= \sum_{r=0}^{7} {}^7C_r \cdot {}^7C_r = ({}^7C_0)^2 + ({}^7C_1)^2 + ({}^7C_2)^2 + .... + ({}^7C_7)^2$$

$= 3432$

**44- 46 :**

**44. (c) :** The natural numbers are written in rows

| | | | | |
|---|---|---|---|---|
| 1 | 4 | 7 | 10 | 13 |
| 2 | 5 | 8 | 11 | 14 |
| 3 | 6 | 9 | 12 | 15 |

$x^3 + y^3$ is divisible by 3 if and only if $x + y$ is divisible by 3. The numbers $x$ and $y$ are taken one from row 1 and other from row 2 or both from row 3. The desired number is
$({}^5C_1 \times {}^5C_1) + {}^5C_2 = 25 + 10 = 35$

**45. (b) :** The numbers are written in rows

| | | |
|---|---|---|
| 1 | 6 | 11 |
| 2 | 7 | 12 |
| 3 | 8 | 13 |
| 4 | 9 | 14 |
| 5 | 10 | 15 |

$x^2 - y^2 = (x + y)(x - y)$ is divisible by 5 $\Rightarrow$ both $x$ and $y$ are from any of these rows or one from row 1 and other from row 4 or one from row 2 and other from row 3 $\Rightarrow$ desired number is
$5 \times {}^3C_2 + 2({}^3C_1)^2 = 15 + 18 = 33$

**46. (c) :** $x^4 - y^4 = (x^2 + y^2)(x - y)(x + y)$ is divisible by 5 $\Rightarrow$ Both $x$ and $y$ are from any one row or one from any one row and the other from other row of the first 4 rows. The desired number is
$5 \times {}^3C_2 + 6({}^3C_1)^2 = 15 + 54 = 69$

**47. A - r; B - r; C - q; D - p**

(A) One Indian wife and one American wife can be selected in ${}^2C_1 \times {}^2C_1$ ways and keeping an unmarried person in between these two wives, the total number of linear arrangements are ${}^2C_1 \times {}^2C_1 \times \underline{|7} \times \underline{|2} = 40320$

(B) Required number of ways $= \underline{|8} = 40320$

(C) Required number of ways
$= \underline{|(7-1)} \times \underline{|2} \times {}^2C_1 \times {}^2C_1 = 5760$

(D) Number of ways in which interviews can be arranged
$= 9 \times {}^8C_2 \times {}^6C_2 \times {}^4C_2 \times {}^2C_2 = 22680$

**48. A - r; B - s; C - p; D - p**

AA, SSS, TT, EE, RR, M, B, L

(A) Take RAAT as one unit. Therefore 10 + 1 = 11 units can be arranged in $\frac{11!}{3! \times 2!}$ ways.

(B) After fixing S in first position and R in last position, the remaining 12 letters can be arranged in remaining 12 positions in $\frac{12!}{(2!)^4}$ ways.

(C) First 7 positions can be filled with A's, E's, M, B, L in $\frac{7!}{(2!)^2}$ ways. The remaining 7 positions can be filled with S's, T's, R's in $\frac{7!}{3!\times(2!)^2}$ ways.

(D) 7 even positions can be filled with A's, S's, R's in $\frac{7!}{(2!)^2\times 3!}$ ways. 7 odd positions can be filled with T's, E's, M, B, L in $\frac{7!}{(2!)^2}$ ways.

**49. A - q,s; B - q,s; C - p; D - r**

(A) Non-negative integral solutions of $R + B + G + Y = 8$ is ${}^{8+4-1}C_{4-1} = {}^{11}C_3$

(B) $D_1 + D_2 + D_3 + D_4 = 9, D_1 \neq 0$

$D_1 - 1 + D_2 + D_3 + D_4 = 8$

${}^{8+4-1}C_{4-1} = {}^{11}C_3$

(C) Select '3' places out of 21 places $= {}^{21}C_3$

(D) $\left[\frac{17}{3}\right]+\left[\frac{17}{3^2}\right]=5+1=6$

**50. (5) :** Since $2009 = 7 \times 7 \times 41$. Therefore, 41 must divide $n^2(n-1)$, which implies that 41 is a factor of either $n$ or $n-1$. In particular, $n \geq 41$. For $n = 41$, neither $n$ nor $(n-1)$ is divisible by 7. For $n = 42$, $n^2$ is divisible by $7^2$, since $n$ is divisible by 7. Therefore, $n = 42$ is the smallest integer.

**51. (6) :** Images of 1,2,3,4,5 are respectively.

7,7,7,8,9; 7,7,8,8,9; 7,7,8,9,9

7,8,8,8,9; 7,8,8,9,9; 7,8,9,9,9

$\therefore$ There are six functions.

**52. (3) :** There are exactly $(n-1)$ subsets of $S$ containing two elements having 1 as least element; exactly $(n-2)$ subsets of $S$ having 2 as least element and so on.

Thus

$$\text{Thus } \sum_{A\in X}\min(A) = (1)(n-1)+2(n-2)+....+(n-1)(1)$$

$$=\sum_{r=1}^{n-1} r(n-r) = {}^{n+1}C_3$$

**53. (9) :** The number of ways in which all the letters are in wrong envelopes

$$=4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)$$

$$=4!\left(\frac{1}{2}-\frac{1}{6}+\frac{1}{24}\right)=12-4+1=9$$

**54. (0) :** ${}^{100}C_{50} = \frac{100!}{50!50!}$

Exponent of 7 in 100 ! = 16

Exponent of 7 in 50! $= \left[\frac{50}{7}\right]+\left[\frac{50}{7^2}\right]=8$

Exponent of 7 in $(50!)^2 = 16$

$\therefore$ Exponent of 7 in ${}^{100}C_{50} = 16 - 16 = 0$

**55. (3) :** 1! +2! + 3! + 4! = 33

5! = 120, 6! = 720, 7! = 5040

8! = 40320, 9! = 362880

Thus the unit digit of 1! + 2! + …. + 9! = 3

Also note that $n!$ is divisible by 100 for all $n \geq 10$.

$\therefore$ Unit digit of 10! + 11! + ….. + 49! = 0

$\therefore$ Unit digit of 1! + 2! + ……+ 49! = 3.

**56. (7) :** For $n = 6$

$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729 < 900$

For $n = 7$

$3 \times 3 \times 3 \times 3\times 3 \times 3 \times 3 = 2187 > 900$

For $n = 8$

Number of $n$-digits formed > 900

Since the least $n$ is required

$\therefore$ $n = 7$.

**57. (5) :** $a + b + c = 21$

$\Rightarrow$ $b + c > a \Rightarrow a + b + c > 2a$

$\Rightarrow$ $2a < 21 \Rightarrow a \leq 10$.

So, $1 \leq a, b, c \leq 10$

The cases when $a > b > c$ are (10, 9, 2), (10, 8, 3), (10, 7, 4), (10, 6, 5), (9, 8, 4), (9, 7, 5) and (8, 7, 6). So, number of cases when $a$, $b$, $c$ are all distinct is $7 \times 3! = 42$.

The cases when $a = b > c$ or $a > b = c$ are (10, 10, 1), (9, 9, 3), (8, 8, 5) and (9, 6, 6). So number of cases when two same and 1 different is $4 \times (3!/2!) = 12$.

The cases when $a = b = c$ is (7, 7, 7). The total number of ordered triplets = 42 + 12 + 1 = 55.

**58. (3) :** $x_1 + x_2 + x_3 = 15$

$0 \leq x_1 \leq 5, 0 \leq x_2 \leq 10, 0 \leq x_3 \leq 15$

$n$ = coeff. of $x^{15}$ in $(1 - x^6)(1 - x^{11})(1 - x^{16})(1 - x)^{-3}$

$n = 66$

$$\Rightarrow \quad \frac{n}{22}=3$$

**59. (4) :** Number of the form $(2n+1)$, $n \in I$ are all odd natural numbers.

Thus we have to find all odd numbers dividing 120.

These numbers are 1, 3, 5, 15.

Hence, number of divisors = 4.

# CONCEPT BOOSTERS

Class XII

## PROBABILITY

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This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

### TERMS AND DEFINITIONS USED IN PROBABILITY

- **Random Experiment**
  An experiment is called random if
  **(i)** all the outcomes of the experiment are known in advance
  **(ii)** the exact outcome of any specific performance of the experiment is unpredictable
  *i.e.* not known in advance.
- **Sample Space**
  A set whose elements represent all possible outcomes of a random experiment is called the sample space and is usually represented by '$S$'.
  Consider the experiment of tossing a die. If we are interested in the number on the top face, then sample space would be $S_1 = \{1, 2, 3, 4, 5, 6\}$.
  If we are interested only in whether the number is even or odd, then sample space is $S_2 = \{\text{even, odd}\}$. Clearly more than one sample space can be used to describe the outcomes of an experiment. In this case '$S_1$' provides more information than '$S_2$'. If we know which element in $S_1$ occurs, we can tell which outcome in $S_2$ occurs; however, knowledge of what happens in $S_2$ in no way helps us to know which element in $S_1$ occurs.
  In general it is desirable to use a sample space that gives the maximum information concerning the outcomes of the experiment.
- **Sample Point**
  Each element of the sample space is called a sample point.
- **Event**
  An event is a subset of the sample space.
- **Simple Event**
  An event is called a simple event, if it is a singleton subset of the sample space $S$.
- **Compound Event**
  A subset of the sample space $S$ which contains more than one element is called a compound event.
- **Equally likely Events**
  A set of events is said to be equally likely if taking into consideration all the relevant factors there is no reason to expect one of them in preference to the others. For example when a fair coin is tossed, the occurrence of a tail or a head are equally likely.
- **Mutually Exclusive Events**
  A set of events is said to be mutually exclusive if the occurrence of one of them precludes the occurrence of any of the remaining events. For example, when we throw a pair of dice, the events "a sum of 5 occurs", "a sum of 7 occurs" and "a sum of 9 occurs" are mutually exclusive. In set theoretic notation, events $A_1, A_2, ...., A_m$ are mutually exclusive if $A_i \cap A_j = \phi$ for $i \neq j$ and $1 \leq i, j \leq m$.
- **Exhaustive Events**
  A system of events is said to be exhaustive if on each performance of the experiment at least one of the events of the system is must to occur. In set theoretic notation, events $A_1, A_2, .... , A_m$ are

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exhaustive if $\bigcup_{i=1}^{m} A_i = S$. For example on throwing of a die, the events {1, 2}, {2, 3, 4}, {5} and {4, 5, 6} form an exhaustive system of events.

## AXIOMATIC DEFINITION OF PROBABILITY

- **Probability with discrete sample space containing equally likely points**

If there are $n$ exhaustive mutually exclusive and equally likely outcomes of an experiment and $m$ of them are favourable to an event $A$, then the probability of the happening of $A$ is equal to $\frac{m}{n}$ and it is denoted by $P(A)$. Clearly $P(A)$ is a non-negative number not greater than unity. So $0 \le P(A) \le 1$.

If probability of happening of an event $A$ is 1, then $A$ is called certain event and if probability of happening of an event $A$ is zero, then $A$ is called impossible event.

- **Odds in Favour and Odds Against an Event**

As a result of an experiment, if $p$ of the outcomes are favourable to an event $E$ and $q$ of the outcomes are against it, then we say that odds are $p$ to $q$ in favour of $E$ or odds are $q$ to $p$ against $E$.

$\therefore$ Odds in favour of an event $E$

$$= \frac{\text{number of favourable cases}}{\text{number of unfavourable cases}} = \frac{p}{q}$$

and odds against an event $E$

$$= \frac{\text{number of unfavourable cases}}{\text{number of favourable cases}} = \frac{q}{p}$$

If odds in favour of an event $E$ are $p : q$ then the probability of the occurrence of that event is $\frac{p}{p+q}$.

Similarly, the probability of the non-occurrence of that event is $\frac{q}{p+q}$.

## SET THEORETIC PRINCIPLES

If '$A$' and '$B$' be any two events of the sample space, then $A \cup B$ would stand for occurrence of atleast one of them and $A \cap B$ stands for simultaneous occurrence of $A$ and $B$. $\bar{A}$ (or $A'$) stands for non–occurrence of $A$. $\bar{A} \cap \bar{B}$ or $(A' \cap B')$ stands for non–occurrence of both $A$ and $B$.

**(i)** If $A$ and $B$ are any two events, then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

**(ii)** If $A$ and $B$ are mutually exclusive, then
$P(A \cup B) = P(A) + P(B)$

**(iii)** If $A$ is any event, then
$P(\bar{A}) = P(A') = 1 - P(A)$

**(iv)** $P(A \cap B') = P(A) - P(A \cap B)$

**(v)** $P$(exactly one of $A$ and $B$ occurs)
$= P(A \cap B') + P(A' \cap B)$
$= P(A) + P(B) - 2P(A \cap B)$
$= P(A \cup B) - P(A \cap B)$

## SOME THEOREMS

**(i)** If $A \subseteq B$, then
(a) $P(A) \le P(B)$ and
(b) $P(B - A) = P(B) - P(A)$

**(ii)** $P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$ and equality holds if and only if events $A_i$, $i = 1, 2, ..., n$ are exclusive.

**(iii)** $P\left(\bigcup_{i=1}^{n} A_i\right) \le 1$ and equality holds if and only if events $A_i$, $i = 1, 2, ...., n$ are exhaustive.

**(iv)** $\sum_{i=1}^{n} P(A_i) = 1$ if events $A_i$, $i = 1, 2, ..., n$ are exclusive and exhaustive.

**Note :** If events $A_1, A_2, ..., A_n$ are exclusive, then $P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i)$. This is called the rule of sum.

## SOME USEFUL FORMULAE

**(i) For two events *A* and *B***

(a) $P$(atleast one out of them)
$= P(A) + P(B) - P(A \cap B)$

(b) $P$(exactly one out of them)
$= P(A) + P(B) - 2P(A \cap B)$

**(ii) For three events *A*, *B* and *C***

(a) $P$(atleast one out of them)
$= P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

(b) $P$(atleast two out of them)
$= P(B \cap C) + P(C \cap A) + P(A \cap B)$
$- 2P(A \cap B \cap C)$

(c) $P$(exactly two out of them)
$= P(B \cap C) + P(C \cap A) + P(A \cap B)$
$- 3P(A \cap B \cap C)$

(d) $P$(exactly one out of them)
$= P(A) + P(B) + P(C) - 2P(B \cap C)$
$- 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

## INDEPENDENT EXPERIMENTS

If two random experiments are performed separately *i.e.*, their outcomes are independent of each other, then the experiments are called independent experiments.

**For example**

(i) Consider the tossing of a coin twice, clearly the outcome for the second toss is not effected by the result of the first toss. So the two tosses are independent random experiments.

(ii) In throwing of a die and a coin together or one after the other are independent experiments.

**Remarks:**

(i) In drawing of two cards, without replacement, from a well-shuffled ordinary pack of 52-playing cards, the two draws are not independent experiments.

(ii) Let $E_1$ and $E_2$ be two independent random experiments. Let $A$ be an event of experiment $E_1$ only and $B$ be an event of experiment $E_2$ only. Then
$P(A$ occurs in $E_1$ and $B$ occurs in $E_2) = P(A) \cdot P(B)$.

For example, if a coin and a die are thrown together and $A = \{H\}$, $B = \{1, 2, 3, 4\}$, then $P$ (head on coin and a number $\leq 4$ on die) $= P(A) \cdot P(B)$

$$= \frac{1}{2} \times \frac{4}{6} = \frac{1}{3}.$$

- **Binomial Distribution for Successive Events**

If an experiment is repeated '$n$' times and probability of occurrence of an event of one trial is $p$, then

**(i)** Probability of occurring the event exactly '$r$' times is ${}^nC_r\, p^r (1-p)^{n-r}$.

**(ii)** Probability of occurring the event atleast '$r$' times is $\sum_{k=r}^{n} {}^nC_k p^k (1-p)^{n-k}$.

**(iii)** Probability of occurring the event atmost '$r$' times is $\sum_{k=0}^{r} {}^nC_k p^k (1-p)^{n-k}$.

For example if a die is thrown five times and we want the probability of occurrence of a composite number four times, then we have

$$p = \frac{2}{6} = \frac{1}{3},\ q = 1 - \frac{1}{3} = \frac{2}{3} \text{ and } n = 5,\ r = 4.$$

Thus required probability $= {}^nC_r\, p^r\, q^{n-r}$

$$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 = \frac{10}{243}$$

## CONDITIONAL PROBABILITY

The probability of occurrence of an event $B$ when it is known that some event $A$ has already occurred, is called the conditional probability and is denoted by $P(B|A)$. The symbol $P(B|A)$ is usually read as probability of $B$, given $A$. Consider two events $A$ and $B$. When it is known that event '$A$' has occurred, it means that sample space would reduce to that sample points representing event $A$. Now for $P(B|A)$, we must look for the sample points representing the simultaneous occurrence of $A$ and $B$ *i.e.*, sample points in $A \cap B$.

$$\text{Now, } P(B|A) = \frac{n(A \cap B)}{n(A)} = \frac{\frac{n(A \cap B)}{n(S)}}{\frac{n(A)}{n(S)}} = \frac{P(A \cap B)}{P(A)}$$

$$\text{Thus, } P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ where } 0 < P(A) \leq 1.$$

$$\text{Similarly, } P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ where } 0 < P(B) \leq 1$$

$$\text{Hence, } P(A \cap B) = \begin{cases} P(A) \cdot P(B|A), & P(A) > 0 \\ P(B) \cdot P(A|B), & P(B) > 0 \end{cases}$$

## INDEPENDENT EVENTS

- **Pairwise Independent Events**

Two events $A$ and $B$ are said to be independent if occurrence or non–occurrence of one does not affect the occurrence or non–occurrence of the other.

*i.e.*, $P(B|A) = P(B)$, $P(A) \neq 0$.

$$\Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B)$$

$$\Rightarrow P(A \cap B) = P(A) \cdot P(B)$$

If the events are not independent, then they are said to be dependent.

- **Mutually independent Events**

Three events $A$, $B$ and $C$ are said to be mutually independent if,

$P(A \cap B) = P(A) \cdot P(B)$, $P(A \cap C) = P(A) \cdot P(C)$,
$P(B \cap C) = P(B) \cdot P(C)$
and $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

These events would said to be pair–wise independent if,

$P(A \cap B) = P(A) \cdot P(B)$, $P(B \cap C) = P(B) \cdot P(C)$ and $P(A \cap C) = P(A) \cdot P(C)$.

Thus, mutually independent events are pair–wise independent but the converse may not be true.

- **Rule of Multiplication**

Suppose $A_1, A_2, ..., A_n$ be $n$ events such that $P(A_1 \cap A_2 \cap .... \cap A_n) \neq 0$, then

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | (A_1 \cap A_2)) \ldots P(A_n | (A_1 \cap A_2 \cap .... \cap A_{n-1}))$$

## TOTAL PROBABILITY THEOREM

Suppose $A_1, A_2 ..., A_n$ are mutually exclusive and exhaustive events, then for any event $B$, we can write $\bigcup_{i=1}^{n} (B \cap A_i) \Rightarrow P(B) = \sum_{i=1}^{n} P(B \cap A_i)$ as events $B \cap A_i$, $i = 1, 2, ..., n$ are exclusive.

$$\Rightarrow P(B) = \sum_{i=1}^{n} P(B | A_i) \cdot P(A_i)$$

## BAYES' THEOREM

Suppose $A_1, A_2, ...., A_n$ are mutually exclusive and exhaustive set of events.

Then the conditional probability that $A_j$ happens (given that $B$ has happened) is given by

$$P(A_j | B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(A_j) \cdot P(B | A_j)}{\sum_{j=1}^{n} P(A_j) \cdot P(B | A_j)},$$

where $j = 1, 2, ....., n$.

# PROBLEMS

## SECTION-I

### Single Correct Answer Type

**1.** If $a$ and $b$ are chosen randomly from the set consisting of numbers 1, 2, 3, 4, 5, 6 with replacement, then probability that $\lim_{x \to 0} \left(\frac{a^x + b^x}{2}\right)^{2/x} = 6$ is

(a) 1/3 (b) 1/4 (c) 1/9 (d) 2/9

**2.** A coin is tossed 7 times. Then the probability that at least 4 consecutive heads appear is

(a) 3/16 (b) 5/32 (c) 5/16 (d) 1/8

**3.** A fair coin is tossed 5 times then probability that two heads do not occur consecutively (No two heads come together) is

(a) $\frac{1}{16}$ (b) $\frac{15}{32}$ (c) $\frac{13}{32}$ (d) $\frac{7}{16}$

**4.** A man throws a die until he gets a number bigger than 3. The probability that he gets 5 in the last throw is

(a) 1/3 (b) 1/4 (c) 1/6 (d) 1/36

**5.** A randomly selected year is containing 53 Mondays then probability that it is a leap year, is

(a) 2/5 (b) 3/5 (c) 4/5 (d) 1/5

**6.** $A$ and $B$ are events such that $P(A) = 0.3$, $P(A \cup B) = 0.8$. If $A$ and $B$ are independent then $P(B) =$

(a) 1/7 (b) 3/7 (c) 5/7 (d) 6/7

**7.** If the letters of the word MATHEMATICS are arranged arbitrarily, the probability that C comes before E, E before H, H before I and I before S is

(a) $\frac{1}{75}$ (b) $\frac{1}{24}$ (c) $\frac{1}{120}$ (d) $\frac{1}{720}$

**8.** A signal which can be green or red with probability $\left(\frac{4}{5}\right)$ and $\left(\frac{1}{5}\right)$ respectively is received by the station $A$ and transmitted to $B$. The probability that each station receive signal correctly = 3/4. If signal received by $B$ is green. The probability that original signal was green is

(a) 3/5 (b) 6/7 (c) 20/23 (d) 9/20

**9.** Four identical oranges and six distinct apples (each a different variety) are distributed randomly into five distinct boxes. The probability that each box gets a total of two objects is

(a) $\frac{813}{109375}$ (b) $\frac{162}{21875}$

(c) $\frac{323}{43750}$ (d) $\frac{151}{21875}$

**10.** A bag contains two red balls and two green balls. A person randomly pulls out a ball, replacing it with a red ball regardless of the colour.What is the probability that all the balls are red after three such replacement ?

(a) 3/8 (b) 7/16 (c) 5/32 (d) 9/32

**11.** Three fair coins are tossed simultaneously. Let $E$ be the event of getting three heads or three tails, $F$ be the event of atleast two heads and $G$ be the event of atmost two heads then which of the following is true?

(a) $P(E \cap F) = P(E) \cdot P(F)$
(b) $P(E \cap G) = P(E) \cdot P(G)$
(c) $P(F \cap G) = P(F) \cdot P(G)$
(d) none of these

**12.** Consider all the 3 digit numbers $abc$ (where $a \neq 0$). If a number is selected at random then the probability that the number is such that $a + b + c = 6$ is

(a) $\frac{2}{15}$ (b) $\frac{7}{75}$ (c) $\frac{7}{600}$ (d) $\frac{7}{300}$

**13.** If three numbers are chosen randomly from the set $\{1, 3, 3^2, ..., 3^n\}$ without replacement then the probability that they form an increasing geometric progression is

(a) $\frac{3}{2n}$, if $n$ is odd (b) $\frac{3}{2n}$, if $n$ is even

(c) $\frac{3n}{n^2-1}$, if $n$ is even (d) $\frac{3n}{2(n^2-1)}$, if $n$ is odd

**14.** A fair coin is tossed until one of the two sides occurs twice in a row. The probability that the number of tosses required is even is

(a) 1/3 (b) 2/3 (c) 1/4 (d) 3/4

**15.** A determinant is chosen at random from the set of all determinants of order 2 with elements 1 and 0 only. The probability that the value of the determinant is positive is

(a) 1/8 (b) 3/16 (c) 5/16 (d) 1/16

**16.** Two numbers $x$ and $y$ are chosen at random from the set $\{1, 2, 3, ...., 3n\}$. The probability that $x^2 - y^2$ is divisible by 3 is

(a) $\frac{5n-3}{3(3n-1)}$ (b) $\frac{2n}{3(3n-1)}$

(c) $\frac{5n}{3(3n-1)}$ (d) $\frac{n}{3n-1}$

**17.** $A$ and $B$ throw alternatively with a pair of dice. $A$ wins if he throws a sum 6 before $B$ throws 7 and $B$ wins if he throws a sum 7 before $A$ throws sum 6. If $A$ starts the game, his chance of winning is

(a) $\frac{30}{61}$ (b) $\frac{31}{61}$ (c) $\frac{15}{61}$ (d) $\frac{60}{61}$

**18.** An unbiased coin is tossed 12 times .The probability that at least 7 consecutive heads show up is

(a) $\frac{1}{128}$ (b) $\frac{1}{64}$ (c) $\frac{9}{256}$ (d) $\frac{7}{256}$

**19.** 10 apples are distributed at random among 6 persons. The probability that atleast one of them will receive none is

(a) $\frac{6}{143}$ (b) $\frac{{}^{14}C_4}{{}^{15}C_5}$ (c) $\frac{137}{143}$ (d) $\frac{143}{137}$

**20.** A certain type of missile hits its target with probability 0.3. The number of missiles that should be fired so that there is atleast an 80% probability of hitting a target is

(a) 3 (b) 4 (c) 5 (d) 6

**21.** From a bag containing 10 distinct balls, 6 balls are drawn simultaneously and replaced. Then 4 balls are drawn. The probability that exactly 3 balls are common to the drawings is

(a) 8/21 (b) 6/19 (c) 5/24 (d) 9/22

**22.** The probability that in a family of 5 members, exactly two members have birthday on Sunday is

(a) $\frac{12\times5^3}{7^5}$ (b) $\frac{10\times6^3}{5^7}$

(c) $\frac{12\times6^2}{5^7}$ (d) $\frac{10\times6^3}{7^5}$

**23.** If $a$ is an integer lying in the closed interval $[-5, 30]$, then the probability that the graph of $y = x^2 + 2(a + 4)x - 5a + 64$ is strictly above the $x$-axis is

(a) 2/9 (b) 1/6 (c) 1/2 (d) 5/9

**24.** The probabilities of $A$, $B$, $C$ solving a problem independently are respectively $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$. If 21 such problems are given to $A$, $B$ and $C$, then the probability that atleast 11 problems can be solved by them is

(a) ${}^{21}C_{11}\left(\frac{1}{2}\right)^{11}$ (b) $\frac{1}{2}$

(c) $\left(\frac{1}{2}\right)^{11}$ (d) ${}^{21}C_{11}\frac{2^{11}}{3^{21}}$

**25.** In a test student either guesses or copies or knows the answer of a multiple choice questions with four choices in which exactly one choice is correct. The probability that he makes a guess is 1/3; The probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it is 1/8. Find the probability that he knew the answer to the question given that he correctly answered it is

(a) $\frac{29}{35}$ (b) $\frac{24}{29}$ (c) $\frac{1}{7}$ (d) $\frac{1}{9}$

## SECTION-II

### Multiple Correct Answer Type

**26.** Let us define the events $A$ and $B$ as

$A$ : An year chosen at random contains 29 February.

$B$ : An year chosen at random has 52 Fridays.

If $P(E)$ denotes the probability of happening of event $E$, then

(a) $P(\bar{B}) = \frac{2}{7}$ (b) $P(B) = \frac{23}{28}$

(c) $P(A \mid \bar{B}) = \frac{2}{5}$ (d) $P(A \mid B) = \frac{5}{23}$

**27.** $A$, $B$ are two events of a random experiment such that $P(\bar{A}) = 0.3$, $P(B) = 0.4$ and $P(A \cap \bar{B}) = 0.5$. Then

(a) $P(A \cup B) = 0.9$ (b) $P(B \cap \bar{A}) = 0.2$

(c) $P(\bar{A} \cup \bar{B}) = 0.8$ (d) $P(B \mid (A \cup \bar{B})) = 0.25$

**28.** Contents of the two urns is as given in this table. A fair die is tossed. If the face 1, 2, 4 or 5 comes, a marble is drawn from the urn $A$ otherwise a marble is chosen from the urn $B$.

| Urn | Red marbles | White marbles | Blue marbles |
|---|---|---|---|
| $A$ | 5 | 3 | 8 |
| $B$ | 3 | 5 | 0 |

Let $E_1$ : Denote the event that a red marble is chosen.

$E_2$ : Denote the event that a white marble is chosen.

$E_3$ : Denote the event that a blue marble is chosen.

Then,

(a) the event $E_1$, $E_2$ and $E_3$ are equiprobable.

(b) $P(E_1)$, $P(E_2)$, $P(E_3)$ are in A.P.

(c) if the marble drawn is red, the probability that it came from the urn $A$ is 1/2.

(d) if the marble drawn is white, the probability that the face 5 appeared on the die is 3/32.

**29.** A random variable $x$ takes values 0, 1, 2, 3, ... with probability proportions to $(x+1)\left(\frac{1}{5}\right)^x$, then

(a) $P(x=0) = \frac{16}{25}$ (b) $P(x \le 1) = \frac{112}{125}$

(c) $P(x \ge 1) = \frac{9}{25}$ (d) none of these

**30.** If $A$ and $B$ are two independent events such that $P(A' \cap B) = 2/15$ and $P(A \cap B') = 1/6$, then $P(B)$ can be

(a) 1/5 (b) 1/6 (c) 4/5 (d) 5/6

**31.** If $M$ and $N$ are any two events, the probability that the exactly one of them occurs is

(a) $P(M) + P(N) - 2P(M \cap N)$

(b) $P(M) + P(N) - P(M \cap N)$

(c) $P(M^c) + P(N^c) - 2P(M^c \cap N^c)$

(d) $P(M \cap N^c) + P(M^c \cap N)$

**32.** A bag initially contains one red ball and two blue balls. An experiment consists of selecting a ball at random, noting its colour and replacing it together with an additional ball of the same colour. If three such trials are made, then

(a) Probability that atleast one blue ball is drawn is 0.9

(b) Probability that exactly one blue ball is drawn is 0.2

(c) Probability that all the drawn balls are red given that all the drawn balls are of the same colour is 0.2

(d) Probability that atleast one red ball is drawn is 0.6

**33.** The probability that exactly one of the independent events $A$ and $B$ occurs is equal to

(a) $P(A) + P(B) - 2P(A \cap B)$

(b) $P(A) + P(B) - P(A \cap B)$

(c) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A} \cap \bar{B})$

(d) $P(A) + P(B) - 3P(A \cap B)$

**34.** If $P(A) = 3/5$ and $P(B) = 2/3$, then

(a) $P(A \cup B) \ge \frac{2}{3}$ (b) $\frac{4}{15} \le P(A \cap B) \le \frac{3}{5}$

(c) $P(A \cap \bar{B}) \le \frac{1}{3}$ (d) $P(A \cup B) \ge \frac{3}{5}$

## SECTION-III

### Comprehension Type

**Paragraph for Question No. 35 to 37**

A lot contains 10 defective and 10 non-defective bulbs. 2 bulbs are drawn at random, one at a time with replacement. We define the events $A$, $B$ and $C$ as follows:

$A$ = {The first bulb is defective}.

$B$ = {The second bulb is non-defective}.

$C$ = {Both bulbs are either defective or non-defective}.

**35.** $P(A)$ will be equal to

(a) 1/4 (b) 3/4 (c) 1/2 (d) 1/3

**36.** $P(B) \cdot P(C)$ will be equal to

(a) 1/4 (b) 1/2 (c) 1/16 (d) 1/8

**37.** $P(A \cap B \cap C)$ will be equal to

(a) 0 (b) 1/2 (c) 1/4 (d) 1/8

**Paragraph for Question No. 38 to 40**

Let $S$ be the set of first 18 natural numbers. Then

**38.** The probability of choosing $\{x, y\} \subseteq S$ such that $x^3 + y^3$ is divisible by 3 is

(a) 1/3 (b) 1/6 (c) 1/5 (d) 1/4

**39.** The probability of choosing $\{x, y, z\} \subseteq S$ such that $x$, $y$, $z$ are in A.P. is

(a) 1/17 (b) 2/17 (c) 5/34 (d) 3/34

**40.** The probability of choosing $\{x, y, z\} \subseteq S$ such that no two of the numbers $x$, $y$, $z$ are consecutive is

(a) 35/51 (b) 2/17 (c) 4/17 (d) 6/17

## SECTION-IV

### Matrix-Match Type

**41.** '$n$' whole numbers are randomly chosen and multiplied.

| Column-I | | Column-II | |
|---|---|---|---|
| A. | The probability that the last digit is 1, 3, 7 or 9 is | p. | $\frac{8^n-4^n}{10^n}$ |
| B. | The probability that the last digit is 2, 4, 6, 8 is | q. | $\frac{5^n-4^n}{10^n}$ |
| C. | The probability that the last digit is 5 is | r. | $\frac{4^n}{10^n}$ |
| D. | The probability that the last digit is zero is | s. | $\frac{10^n-8^n-5^n+4^n}{10^n}$ |

**42.** Match the functions given in Column I with their domain in Column II. $A$ is a set containing $n$ elements. A subset $P$ of $A$ is chosen at random. The set $A$ is reconstructed by replacing the elements of the subset $P$. A subset $Q$ of $A$ is again chosen at random, then the probability that

| Column-I | | Column-II | |
|---|---|---|---|
| A. | $P \cap Q = \phi$ | p. | $\frac{n(3^{n-1})}{4^n}$ |
| B. | $P \cap Q$ is a singleton | q. | $(3/4)^n$ |
| C. | $P \cap Q$ contain 2 elements | r. | $\frac{{}^{2n}C_n}{4^n}$ |
| D. | $\|P\| = \|Q\|$, where $\|X\|$ = number of elements in $X$ | s. | $\frac{3^{n-2}(n-1)n}{2(4^n)}$ |

**43.** There are 10 pairs of shoes in a cupboard from which 4 shoes are taken at random. If $P(E)$ denotes the probability of the event $E$.
Match the following:

| Column-I | | Column-II | |
|---|---|---|---|
| A. | $P$ (getting no pair) | p. | $\frac{99}{323}$ |
| B. | $P$ (getting atleast one pair) | q. | $\frac{96}{323}$ |
| C. | $P$ (getting exactly two pairs) | r. | $\frac{224}{323}$ |
| D. | $P$ (getting exactly one pair) | s. | $\frac{3}{323}$ |

**44.** Five unbiased cubical dice are rolled simultaneously. Let $m$ and $n$ be the smallest and the largest number appearing on the upper faces of the dice, then match the following event given in Column I with their respective probabilities given in Column II.

| Column-I | | Column-II | |
|---|---|---|---|
| A. | $m = 3$ | p. | $\left(\frac{2}{3}\right)^5$ |
| B. | $n = 4$ | q. | $\left(\frac{2}{3}\right)^5+\left(\frac{1}{3}\right)^5-\left(\frac{1}{2}\right)^4$ |
| C. | $2 \le m \le 4$ | r. | $\left(\frac{5}{6}\right)^5-\left(\frac{1}{3}\right)^5$ |
| D. | $m = 2$ and $n = 5$ | s. | $\left(\frac{2}{3}\right)^5-\left(\frac{1}{2}\right)^5$ |

## SECTION-V

### Integer Answer Type

**45.** If $n(X) = (k+1)$, then the probability of selecting 2 subsets $A$ and $B$ of the set '$X$' such that $B = A^c$ is equal to $\frac{1}{2^{m-1}}$, where $m - k$ is equal to

**46.** A bag contains 10 different balls. Five balls are drawn simultaneously and then replaced and then seven balls are drawn. The probability that exactly three balls are common to the two drawn is $p$, then the value of $12p$ is

**47.** If the sides of a triangle are decided by throwing a die thrice, the probability that the triangle is isosceles or equilateral is $1/k$, then $k =$

**48.** Four identical dice are rolled once, the probability that all the numbers on them are primes is $\frac{L}{8L+2}$, then $L =$

**49.** If $\{x, y\}$ is a subset of the first 30 natural numbers, then the probability that $x^3 + y^3$ is divisible by 3, is $S/9$ then $S =$

**50.** An unbiased coin is tossed. If the result is a head, a pair of unbiased dice is rolled and the number obtained by adding the numbers on the two faces is noted. If the result is a tail, a card from a well shuffled pack of eleven cards numbered 2, 3, 4, 5, ...., 12 is picked and the number on the card is noted. The probability that the noted number is either 7 or 8 is $\frac{P}{792}$, then the digit in tens place of $P$ is

**51.** 1, $\alpha_1$, $\alpha_2$, $\alpha_3$, ..., $\alpha_k$ are divisors of number $N = 2^{n-1}(2^n - 1)$, where $2^n - 1$ is a prime number and $1 < \alpha_1 < \alpha_2 < \alpha_3 < .... < \alpha_k$, then value of $\left(1+\frac{1}{\alpha_1}+\frac{1}{\alpha_2}+....+\frac{1}{\alpha_k}\right)$ is

**52.** If the integers $m$ and $n$ are chosen at random from {1, 2, 3, ....., 100} then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is equal to $1/k$. The numerical value of $k$ is

**53.** A man parks his car among $n$ cars standing in a row, his car not being parked at an end, on his return he finds that exactly $m$ of the $n$ cars are still there, probability that both the cars parked on two sides of his car, have left is $\frac{(n-m)(n-m-1)}{(n-A)(n-B)}$, then $A + B$ is

**54.** A special die is so constructed that the probabilities of throwing 1, 2, 3, 4, 5 and 6 are $(1 - k)/6$, $(1 + 2k)/6$, $(1 - k)/6$, $(1 + k)/6$, $(1 - 2k)/6$ and $(1 + k)/6$ respectively. If two such dice are thrown and the probability of getting a sum equal to 9 lies in $\left[\frac{1}{9}, \frac{2}{9}\right]$. Then find the number of integral solutions of $k$.

**55.** Two distinct numbers are chosen at random from the set {1, 2, ……. $3n$}. The probability that $x^2 - y^2$ is divisible by 3 is $\frac{pn+q}{r(3n-1)}$, then $p + q + r =$

**56.** 3 numbers are chosen at random without replacement from {1, 2, 3, ...., 14}.
Let $A$ = {min of chosen number is 5}
$B$ = {max of chosen number is 11}
$P(A \cup B) = \frac{K+11}{91}$, then $K =$ _____

**57.** In a bag there are 15 balls of either red or green colour. Let $G_k$ be the event that it contains exactly $k$ green balls and its probability is proportional to $k^2$. Now a ball is drawn at random. Let $P(A)$ be the probability that the ball drawn is green. If $P(A) = p/q$ in lowest form, then $q - p$ is _____.

## SOLUTIONS

**1. (c) :** $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{2/x} = 6$

$= e^{\lim_{x\to 0}\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)} = 6 = e^{\log a + \log b} = 6$

$\Rightarrow ab = 6$

$(a, b) = (1, 6), (6, 1), (2, 3), (3, 2)$

$\therefore$ Required probability $= \frac{4}{6\times 6} = \frac{1}{9}$

**2. (b) :** Let $H$ denote the head, $T$ the tail.
* Any of the head or tail

$P(H) = \frac{1}{2}, \; P(T) = \frac{1}{2}, \; P(*) = 1$

$HHHH** = \left(\frac{1}{2}\right)^4 \times 1 = \frac{1}{16}$

$THHHH** = \left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

$*THHHH* = \left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

$**THHHH = \left(\frac{1}{2}\right)^5 \times 1 = \frac{1}{32}$

$\therefore$ The required probability $= \frac{5}{32}$.

**3. (c) :** $P\left(\frac{E}{\text{no heads}}\right)+P\left(\frac{E}{\text{1 head}}\right)+P\left(\frac{E}{\text{2 heads}}\right)+P\left(\frac{E}{\text{3 heads}}\right)$,

where $E$ : getting no two consecutive heads.

$= \frac{1}{32}+\frac{5}{32}+\frac{6}{32}+\frac{1}{32} = \frac{13}{32}$

**4. (a) :** $P$(getting a number bigger than 3) = 1/2
$P$(getting 5 in throw) = 1/6
Let $E$ : getting 5 in last throw when he gets a number bigger than 3.

$\therefore \quad P(E) = \frac{1}{6}+\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{6}+....\infty$

$= \frac{1}{6}\times\frac{1}{1-\frac{1}{2}} = \frac{1}{3}$

**5. (a) :** Selected year may be a non leap year with a probability 3/4.
Selected year may leap year with a probability 1/4.
Let $E$ be an event that randomly selected year contains 53 Mondays

$P(E) = \frac{3}{4}\times\frac{1}{7}+\frac{1}{4}\times\frac{2}{7} = \frac{5}{28} \Rightarrow P\left(\frac{\text{leap year}}{E}\right) = \frac{\frac{2}{28}}{\frac{5}{28}} = \frac{2}{5}$

**6. (c) :** $P(A \cap B) = P(A) \cdot P(B)$
$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$
$0.8 = 0.3 + P(B)\,(1 - 0.3)$

$0.5 = P(B)(0.7) \Rightarrow P(B) = \frac{5}{7}$

**7. (c) :** The total numbers of arrangements is

$$\frac{11!}{2!2!2!}=\frac{11!}{8}$$

The number of arrangements in which C, E, H, I, S appear in that order required

$$=\binom{11}{5}\frac{6!}{2!2!2!}=\frac{11!}{8\cdot 5!}$$

$\therefore$ Required probability $=\frac{11!}{8\cdot 5!}\div\frac{11!}{8!}=\frac{1}{5!}=\frac{1}{120}$

**8. (c) :** $G$ = Original signal was green. $A$ = $A$ receive correct signal, $B$ = $B$ receive correct signal and $E$ = signal received by $B$ is green.

$$P\left(\frac{G}{E}\right)=\frac{P(G\cap E)}{P(E)}$$

$$P(E)=P(GAB)+P(G\bar{A}\bar{B})+P(\bar{G}A\bar{B})+P(\bar{G}\bar{A}B)$$

$$=\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{3}{4}+\frac{4}{5}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{5}\cdot\frac{1}{4}\cdot\frac{3}{4}=\frac{46}{80}$$

$$P(G\cap E)=P(GAB)+P(G\bar{A}\bar{B})=\frac{40}{80}$$

**9. (b) :** The total number of ways to put four identical oranges and six distinct apples into five distinct boxes is

$$({}^{5+4-1}C_4)\cdot 5^6 = 70\times 5^6$$

To satisfy the criteria that each box contains two object we make three cases

(1) Two oranges in each of the two boxes and no orange in the other three boxes

Number of ways $={}^5C_2\times\frac{6!}{(2!)^3}=900$

(2) Two oranges in one box, one orange in each of the two other boxes $=5\times({}^4C_2)\times\frac{6!}{(2!)^2}=5400$

(3) One orange in each of the four boxes

$$=5\cdot\frac{6!}{2!}=1800$$

The total number of ways that each box contains two objects = 900 + 5400 + 1800 = 8100

$\therefore$ Required probability $=\frac{8100}{70\times 5^6}=\frac{162}{21875}$

**10. (d) :** In order that all balls are red after 3 replacements, two of the three balls selected must have green. There could be three cases.

I : Red, Green, Green; II : Green, red, Green.

III : Green,Green,Red.

The probabilities is

In case I $=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4}=\frac{1}{16}$, In case II $=\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4}=\frac{3}{32}$

In case III $=\frac{1}{2}\cdot\frac{1}{4}\cdot 1=\frac{1}{8}$

The required probability $=\frac{1}{16}+\frac{3}{32}+\frac{1}{8}=\frac{9}{32}$

**11. (a) :** $P(E)=\frac{2}{8}=\frac{1}{4}$, $P(F)=\frac{4}{8}=\frac{1}{2}$, $P(G)=\frac{7}{8}$

$P(E\cap F)=\frac{1}{8}$, $P(F\cap G)=\frac{3}{8}$, $P(E\cap G)=\frac{1}{8}$

**12. (d) :** Since $a+b+c=6$, the possible digit selections are (1, 2, 3), (1, 1, 4), (2, 2, 2), (0, 1, 5), (0, 2, 4), (0, 3, 3), (0, 0, 6)

The required number of ways

= 6 + 3 + 1 + 4 + 4 + 2 + 1 = 21

Required probability $=\frac{21}{9\times 10\times 10}=\frac{7}{300}$

**13. (a) :** Number of triplets $(3^r, 3^{r+1}, 3^{r+2})$ $(0\le r\le n)$ is $n-1$

Number of triplets $(3^r, 3^{r+2}, 3^{r+4})$ $(0\le r\le n)$ is $n-3$

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Number of triplets $\left(3^r, 3^{r+\frac{n-1}{2}}, 3^{r+n-1}\right)(n \text{ odd})$ is 2

And number of triplets $\left(3^r, 3^{r+\frac{n}{2}}, 3^{r+n}\right)(n \text{ even})$ is 1.

$\therefore$ If $n$ is odd, the number of favourable outcomes

$$=(n-1)+(n-3)+\ldots+4+2=\frac{n^2-1}{4}$$

And if $n$ is even, the number of favourable outcomes

$$=(n-1)+(n-3)+\ldots+3+1=\frac{n}{2}\times\frac{n}{2}=\frac{n^2}{4}$$

Probability $=\frac{(n^2-1)/4}{{}^{(n+1)}C_3}=\frac{3}{2n}$ if $n$ is odd

$=\frac{n^2/4}{{}^{(n+1)}C_3}=\frac{3n}{2(n^2-1)}$ if $n$ is even.

**14. (b) :** $A$ = {HH, HTHH, HTHTHH, ....}

and $B$ = {TT, THTT, THTHTT, ...}

$$P(A)=\frac{1}{2^2}+\frac{1}{2^4}+\ldots=\frac{1}{3} \text{ and } P(B)=\frac{1}{3}$$

$\therefore$ Required probability $=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

**15. (b) :** No. of determinants formed = $n(S)=2^4=16$

Let $E$ : the determinants $\begin{vmatrix}1&0\\0&1\end{vmatrix}, \begin{vmatrix}1&0\\1&1\end{vmatrix}, \begin{vmatrix}1&1\\0&1\end{vmatrix}$ are positive

$\Rightarrow$ $n(E)=3$ $\therefore$ $P(E)=\frac{3}{16}$

**16. (a) :** $G_1 = 1 \quad 4 \quad 7 \quad \text{----} \quad 3n-2$
$G_2 = 2 \quad 5 \quad 8 \quad \text{----} \quad 3n-1$
$G_3 = 3 \quad 6 \quad 9 \quad \text{----} \quad 3n$

We have $n(S) = {}^{3n}C_2$, for $n(E)$ both numbers can be selected from same group or one number from $G_1$ and one from $G_2$

$\therefore \quad n(E) = 3 \cdot {}^{n}C_2 + {}^{n}C_1 \times {}^{n}C_1 \quad \therefore \quad P(E) = \dfrac{3 \cdot {}^{n}C_2 + n^2}{{}^{3n}C_2}$

**17. (a) :** A's chance of winning in a throw $= \dfrac{5}{36}$,

$B$'s chance of winning in a throw $= \dfrac{1}{6}$

A's chance of losing in a throw $= \dfrac{31}{36}$,

$B$'s chance of losing in a throw $= \dfrac{5}{6}$

A's chance of winning the game

$$= \frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + \frac{31}{36}\times\frac{5}{6}\times\frac{31}{36}\times\frac{5}{6}\times\frac{5}{36} + \ldots.$$

$$= \frac{5}{36}\left[1 + \frac{155}{216} + \left(\frac{155}{216}\right)^2 + \ldots.\right] = \frac{30}{61}$$

**18. (d) :** The sequence of consecutive heads may start with 1st toss or 2nd toss or 3rd toss .... or at 6th toss. In any case, if it starts with $r^{th}$ throw, the first $(r-2)$ throws may be head or tail but $(r-1)^{st}$ throw must be tail, after which again a head or tail can show up:

$\underbrace{HHH\text{------}H}_{7 \text{ times}} \times\times\times\times\times$

or $\underbrace{\times\times\times\text{-----}\times}_{(r-2)}\ \underset{\downarrow}{T}\ \underbrace{HHH\text{------}H}\times\times\text{-----}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad (r-1)^{st}$

$$\therefore \text{ Probability} = \frac{1}{2^7} + \underbrace{\frac{1}{2}\cdot\frac{1}{2^7} + \frac{1}{2}\cdot\frac{1}{2^7} + \ldots. + \frac{1}{2}\cdot\frac{1}{2^7}}_{5 \text{ times}}$$

$$= \frac{1}{2^7}\left[1 + \frac{5}{2}\right] = \frac{7}{2^8}$$

**19. (c) :** The required probability = 1 – probability of each receiving atleast one $= 1 - \dfrac{n(E)}{n(S)}$.

Now, the number of integral solutions of
$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$
Such that $x_1 \geq 1, x_2 \geq 1, \ldots, x_6 \geq 1$ gives $n(E)$ and the number of integral solutions of $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$ such that $x_1 \geq 0, x_2 \geq 0, \ldots, x_6 \geq 0$ gives $n(S)$

$$\therefore \text{ The required probability} = 1 - \frac{{}^{10-1}C_{6-1}}{{}^{10+6-1}C_{6-1}}$$

$$= 1 - \frac{{}^{9}C_5}{{}^{15}C_5} = \frac{137}{143}$$

**20. (c) :** Let $n$ be the required number.
The probability that '$n$' missiles miss the target is $(0.7)^n$. We require $1 - (0.7)^n \geq 0.8$
*i.e.*, $(0.7)^n \leq 0.2$. The least value of '$n$' satisfying the inequality is 5.

**21. (a) :** Let $S$ be the sample space of the composite experiment of drawing 6 balls in the first draw and then 4 in second draw then $|S| = {}^{10}C_6 \times {}^{10}C_4$

$$\therefore \text{ Required probability} = \frac{{}^{10}C_6 \times {}^{6}C_3 \times {}^{4}C_1}{{}^{10}C_6 \times {}^{10}C_4}$$

$$= \frac{80\times24}{10\times9\times8\times7} = \frac{8}{21}$$

**22. (d) :** Required probability

$$= \frac{{}^{5}C_2 \times 6\times6\times6}{7\times7\times7\times7\times7} = \frac{10\times6^3}{7^5}$$

**23. (c) :** $(a+4)^2 + 5a - 64 < 0 \implies -16 < a < 3$

$$\therefore \text{ Required probability} = \frac{18}{36} = \frac{1}{2}$$

**24. (b) :** No. of trials $n = 21$
Let success of solving the problem be $p$.

$$\therefore \quad p = P(A \cup B \cup C) = 1 - P(\bar{A})P(\bar{B})P(\bar{C})$$

$$= 1 - \frac{3}{4}\times\frac{4}{5}\times\frac{5}{6} = \frac{1}{2}, \quad q = \frac{1}{2}$$

Find $P(X \geq 11)$

**25. (b) :** Let '$A$' be the event of guessing the correct answer. '$B$' be the event of copying the correct answer. '$C$' be the event of knowing the correct answer.
'$D$' be the event that his answer is correct

$$\left.\begin{aligned} P(A) = \frac{1}{3}, P(B) = \frac{1}{6}, P(C) = \frac{1}{2} \\ P\left(\frac{D}{B}\right) = \frac{1}{8}, P\left(\frac{D}{A}\right) = \frac{1}{4}, P\left(\frac{D}{C}\right) = 1 \end{aligned}\right\}$$

$$P\left(\frac{C}{D}\right) = \frac{P(C)\cdot P\left(\frac{D}{C}\right)}{P(D\cap A) + P(D\cap B) + P(D\cap C)} = \frac{24}{29}$$

**26. (b, c, d) :** $P(A) = 1/4$, $P(B \mid A) = 5/7$,
$P(B \mid \bar{A}) = 6/7$, $P(B) = P(A)P(B \mid A) + P(\bar{A})P(B \mid \bar{A})$
$= (1/4 \times 5/7) + (3/4 \times 6/7) = 23/28$

$$P(A \mid B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{1}{4}\times\frac{5}{7}}{\frac{1}{4}\times\frac{5}{7} + \frac{3}{4}\times\frac{6}{7}} = \frac{5}{23}$$

$$P(A \mid \bar{B}) = \frac{P(A\cap\bar{B})}{P(\bar{B})} = \frac{\frac{1}{4}\times\frac{2}{7}}{1 - \frac{23}{28}} = \frac{2}{5}$$

**27. (a, b, c, d) :** $P(A) = 0.7$; $P(B) = 0.4$

$P(A - B) = P(A) - P(AB)$

$\Rightarrow P(AB) = 0.2 \Rightarrow P(A + B) = 0.9$

$\Rightarrow P(B - A) = 0.2$

$\Rightarrow P(\bar{A} \cup \bar{B}) = 1 - P(AB) = 0.8$

$\Rightarrow P(B \mid A \cup \bar{B}) = \dfrac{P(A \cap B)}{P(A \cup \bar{B})} = \dfrac{1}{4}$

**28. (a, b, d) :**

| Urn | Red marbles | White marbles | Blue marbles |
|---|---|---|---|
| $A$ | 5 | 3 | 8 |
| $B$ | 3 | 5 | 0 |

$$P(E_1) = P(R) = \left(\frac{2}{3}\right)\left(\frac{5}{16}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{8}\right) = \frac{10}{48} + \frac{6}{48} = \frac{1}{3}$$

$$P(E_2) = P(W) = \left(\frac{2}{3}\right)\left(\frac{3}{16}\right) + \left(\frac{1}{3}\right)\left(\frac{5}{8}\right) = \frac{6}{48} + \frac{10}{48} = \frac{1}{3}$$

$$P(E_3) = P(B) = \left(\frac{2}{3}\right)\left(\frac{8}{16}\right) = \frac{1}{3}$$

(c) Let $A$ : event that urn $A$ is chosen

$$P(A \mid R) = \frac{P(A \cap R)}{P(R)} = \frac{\left(\frac{2}{3}\right)\left(\frac{5}{16}\right)}{\frac{1}{3}} = \left(\frac{10}{48}\right)(3) = \frac{5}{8}$$

$\Rightarrow$ (c) is incorrect.

(d) $$P(A \mid W) = \frac{P(A \cap W)}{P(W)} = \frac{\left(\frac{2}{3}\right)\left(\frac{3}{16}\right)}{\frac{1}{3}} = \left(\frac{6}{48}\right)(3) = \frac{3}{8}$$

$$P(\text{face five} \mid W) = \left(\frac{3}{8}\right)\left(\frac{1}{4}\right) = \frac{3}{32}$$

$\Rightarrow$ (d) is correct.

**29. (a, b, c) :** We have, $P(X = x) \propto (x+1)\left(\dfrac{1}{5}\right)^x$

since, $\sum\limits_{x=0}^{\infty} P(X = x) = 1$

$$\Rightarrow k\left\{1 + 2\left(\frac{1}{5}\right) + 3\left(\frac{1}{5}\right)^2 + 4\left(\frac{1}{5}\right)^3 + \ldots \infty\right\} = 1$$

$$\Rightarrow k = \frac{16}{25}$$

(a) $$P(X = 0) = k(0+1)\left(\frac{1}{5}\right)^0 = k = \frac{16}{25}$$

(b) $P(X \le 1) = P(X = 0) + P(X = 1) = \dfrac{112}{125}$

(c) $P(X \ge 1) = 1 - P(X = 0) = 1 - k = 1 - \dfrac{16}{25} = \dfrac{9}{25}$

**30. (b, c) :** Since $A$ and $B$ are independent events,

$\dfrac{2}{15} = P(A' \cap B) = P(A')\,P(B)$

$= [1 - P(A)]\,P(B)$ ...(1)

and $\dfrac{1}{6} = P(A \cap B') = P(A)\,P(B')$

$= P(A)[1 - P(B)]$ ... (2)

Subtracting (1) from (2), we get

$P(A) - P(B) = 1/30$ or $P(A) = P(B) + 1/30$

Putting this value in (2), we get

$[P(B) + 1/30][1 - P(B)] = 1/6$

$\Rightarrow 30[P(B)]^2 - 29\,P(B) + 4 = 0$

$\Rightarrow P(B) = 1/6, 4/5$

**31. (a, c, d) :** The required probability

= prob. that $M$ occurs and $N$ does not occur or $N$ occurs and $M$ does not occur.

$= P(M \cap N^c) + P(M^c \cap N)$ [This is (d)]

$= P(M) - P(M \cap N) + P(N) - P(M \cap N)$

$= P(M) + P(N) - 2P(M \cap N)$ [This is (a)]

$= 1 - P(M^c) + 1 - P(N^c) - 2[1 - P(M \cap N)^c]$

$= 2P(M^c \cup N^c) - P(M^c) - P(N^c)$

[By De-Morgan's law]

$= 2[P(M^c) + P(N^c) - P(M^c \cap N^c] - P(M^c) - P(N^c)]$

$= P(M^c) + P(N^c) - 2P(M^c \cap N^c)$ [This is (c)]

**32. (a, b, c, d) :** Probability that atleast one blue ball is drawn

= 1 – Probability that all the balls drawn are red.

$$= 1 - \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{3}{5} = 1 - \frac{1}{10} = \frac{9}{10} = 0.9$$

Probability that exactly one blue ball is drawn

$$= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{5} + \frac{2}{3}\cdot\frac{1}{4}\cdot\frac{2}{5} + \frac{1}{3}\cdot\frac{2}{4}\cdot\frac{2}{5} = 0.2$$

Probability that all the drawn balls are red given that all the drawn balls are of the same colour

$$= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{4}{10}} = \frac{1}{5} = 0.2$$

Probability that atleast one red ball is drawn

$$= 1 - \left(\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\right) = 0.6$$

**33. (a, c) :** The probability of exactly one of $A$ and $B$ to occur $= P(A\bar{B}) + P(\bar{A}B)$

$= P(A)P(\bar{B}) + P(\bar{A})P(B) = P(A)\{1 - P(B)\} + \{1 - P(A)\}P(B)$

$= P(A) + P(B) - 2P(A)P(B)$

$= P(A) + P(B) - 2P(A \cap B)$

$= 1 - P(\bar{A}) + 1 - P(\bar{B}) - 2\{1 - P(\bar{A})\}\{1 - P(\bar{B})\}$

$= P(\bar{A}) + P(\bar{B}) - 2P(\bar{A}) \cdot P(\bar{B})$

**34. (a, b, c, d) :** $P(A \cup B) \geq P(A)$, $P(A \cup B) \geq P(B)$

$$P(A \cup B) \leq 1 \Rightarrow \frac{3}{5}+\frac{2}{3}-1 \leq P(A \cap B)$$

$$\Rightarrow \quad P(A \cap B) \geq \frac{4}{15}$$

But $P(A \cap B) \leq P(A)$, $P(A \cap B) \leq P(B)$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B) \leq \frac{3}{5} - \frac{4}{15} \leq \frac{1}{3}$$

**35. (c)** **36. (a)** **37. (a)**

**38. (a) :**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 4 | 7 | 10 | 13 | 16 |
| 2 | 5 | 8 | 11 | 14 | 17 |
| 3 | 6 | 9 | 12 | 15 | 18 |

$$\text{Probability} = \frac{{}^6C_2 + ({}^6C_1)^2}{{}^{18}C_2} = \frac{1}{3}$$

**39. (d) :**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |

$$\text{Probability} = \frac{2({}^9C_2)}{{}^{18}C_3} = \frac{3}{34}$$

**40. (a) :** $\text{Probability} = \dfrac{{}^{(18-3+1)}C_3}{{}^{18}C_3} = \dfrac{35}{51}$

**41. A → r; B → p: C → q; D → s**

(A) The required event will occur if last digit in all chosen numbers is 1, 5, 7 or 9.

$$\text{Required probability} = \left(\frac{4}{10}\right)^n$$

(B) The required probability is equal to the probability that last digit is 2, 4, 6, 8

$$P(1,2,3,4,5,6,7,8,9) - P(1,3,7,9) = \frac{8^n - 4^n}{10^n}$$

(C) $$P(1,3,5,7,9) - P(1,3,7,9) = \frac{5^n - 4^n}{10^n}$$

(D) $$P(0,\ 5) - P(5) = \frac{(10^n - 8^n) - (5^n - 4^n)}{10^n}$$

$$= \frac{10^n - 8^n - 5^n + 4^n}{10^n}$$

**42. A → q; B → p; C → s; D → r**

If $x_i \in A$ then $x_i \in P, x_i \in Q$

or $x_i \notin P, x_i \notin Q$ or $x_i \notin P, x_i \in Q$ or $x_i \in P, x_i \notin Q$

(A) $P \cap Q = \phi \Rightarrow x_i \in P, x_i \notin Q$

or $x_i \notin P, x_i \notin Q$ or $x_i \notin P, x_i \in Q$ or $n(E) = 3^n, n(S) = 4^n$

(B) $P \cap Q$ is a singleton set.

$\Rightarrow x_i \in P, x_i \in Q$

$n(E) = {}^nC_1\ (1)3^{n-1}, n(S) = 4^n$

(C) $P \cap Q$ contain 2 elements

$n(E) = {}^nC_2(1)^2\ 3^{n-2}$

(D) $n(P) = n(Q)$

$$\Rightarrow \quad P(E) = \frac{{}^nC_0\ {}^nC_0 + {}^nC_1\ {}^nC_1 + \ldots + {}^nC_n\ {}^nC_n}{2^n \cdot 2^n}$$

**43. A → r; B → p; C → s; D → q**

(A) $P$ (no pair) $= \dfrac{20}{20} \cdot \dfrac{18}{19} \cdot \dfrac{16}{18} \cdot \dfrac{14}{17} = \dfrac{224}{323}$

(B) $P$ (atleast one pair) $= 1 - \dfrac{224}{323} = \dfrac{99}{323}$

(C) $P$ (exactly two pairs) $= \dfrac{{}^{10}C_2}{{}^{20}C_4} = \dfrac{3}{323}$

(D) $P$ (exactly one pair) $= 1 - \left[\dfrac{224}{323} + \dfrac{3}{323}\right] = \dfrac{96}{323}$

**44. A → s; B → s; C → r; D → q**

(A) The number appearing on upper face of any dice can be 3, 4, 5 or 6 *i.e.* maximum 4 cases.

$P(m = 3) = P(m \geq 3) - P(m \geq 4)$

$$= \frac{4^5 - 3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$$

(B) The number appearing on upper face of any dice can be 1, 2, 3 or 4 *i.e.* maximum 4 cases.

$P(n = 4) = P(n \leq 4) - P(n \leq 3)$

$$= \frac{4^5 - 3^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^5$$

(C) $P(2 \leq m \leq 4) = P(m \geq 2) - P(m \geq 5)$

$$= \frac{5^5 - 2^5}{6^5} = \left(\frac{5}{6}\right)^5 - \left(\frac{1}{3}\right)^5$$

(D) $P(m = 2, n = 5) = P(2, 3, 4 \text{ or } 5) - P(2, 3 \text{ or } 4)$
$\quad - P(3, 4 \text{ or } 5) + P(3 \text{ or } 4)$

$$= \frac{4^5 - 2 \times 3^5 + 2^5}{6^5} = \left(\frac{2}{3}\right)^5 - \left(\frac{1}{2}\right)^4 + \left(\frac{1}{3}\right)^5$$

**45. (2) :** $n(X) = k + 1$

No. of ways to construct $A = 2^{k+1}$

No. of ways to construct $B = 2^{k+1}$

$\therefore$ Total ways to construct $A$ and $B = 2^{k+1} \times 2^{k+1}$

Favourable ways to construct $A = 2^{k+1}$

Favourable ways to construct $B$ such that $B = A^c$ is 1

$\therefore$ Favourable ways $= 2^{k+1} \times 1$

$$\text{Required Probability} = \frac{2^{k+1}}{(2^{k+1})^2} = \frac{1}{2^{k+1}}$$

$\Rightarrow m - 1 = k + 1 \Rightarrow m - k = 2$

# PMT/PET के Entrance Exam में हिंदी माध्यम छात्रों के लिए Triple धमाका

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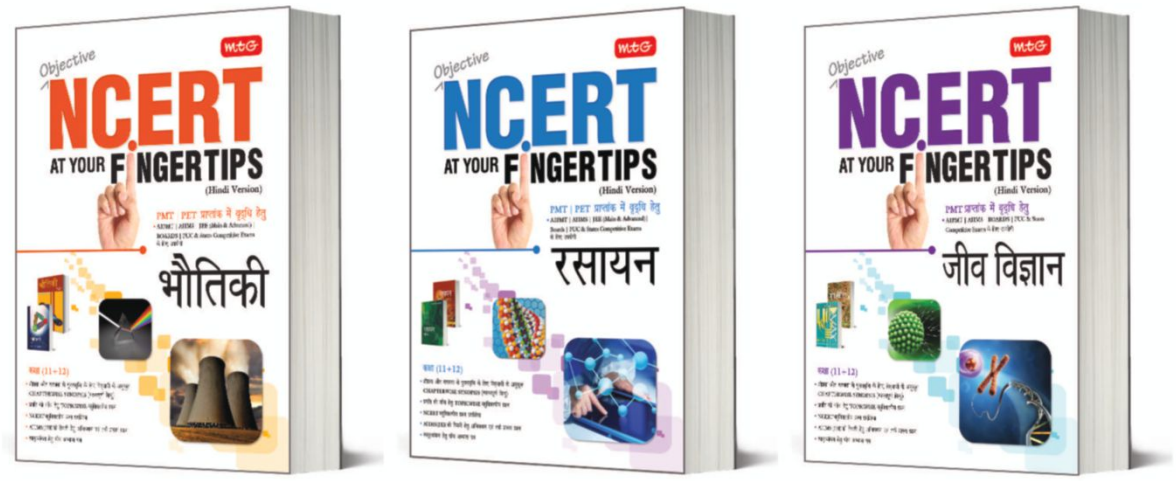


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- प्रश्नों को शीघ्रता और सरलता से हल करने के लिए CHAPTERWISE SYNOPSIS (मुख्य बिंदु)
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**46. (5) :** The no. of ways of drawing 5 balls = $^{10}C_5$
For each set of 7 balls of the second draw, 3 must be common to the set of 5 balls of the draw, *i.e.*, 2 other balls can be drawn in $^3C_2$ ways thus, for each set of 7 balls of the second draw, there are $^7C_3 \times {}^3C_2$ ways of making the first draw so that there are 3 balls common.
Hence, the probability of having three balls in common

$$= \frac{^7C_3 \times {}^3C_2}{^{10}C_5} = \frac{5}{12}.$$

**47. (8) :** Let the sides be $a, b, c$

$a = b = 1, c = 1$
$a = b = 2, c = 1, 2, 3$
$a = b = 3, c = 1, 2, 3, 4, 5$
$a = b = 4, c = 1, 2, 3, 4, 5, 6$
$a = b = 5, c = 1, 2, 3, 4, 5, 6$
$a = b = 6, c = 1, 2, 3, 4, 5, 6$

The number of these triangles is

$1 + 3 + 5 + (3 \times 6) = 27$

$$\text{Probability} = \frac{27}{6^3} = \frac{1}{8}$$

**48. (5) :** The total number of outcomes:

*aaaa* appear in $\binom{6}{1} = 6$ ways

*aaab* appear in $2\binom{6}{2} = 30$ ways

*aabb* appear in $\binom{6}{2} = 15$ ways

*aabc* appear in $3\binom{6}{3} = 60$ ways

*abcd* appear in $\binom{6}{4} = 15$ ways

Total number of ways = 6 + 30 + 15 + 60 + 15 = 126
The number of ways of primes appearing

*aaaa* appear in $\binom{3}{1} = 3$ ways

*aaab* appear in $2\binom{3}{2} = 6$ ways

*aabb* appear in $\binom{3}{2} = 3$ ways

*aabc* appear in $3\binom{3}{3} = 3$ ways number

Total number of ways = 3 + 6 + 3 + 3 = 15

$$\therefore \quad \text{Probability} = \frac{15}{126} = \frac{5}{42}$$

**49. (3) :** $x^3 + y^3$ is divisible by $3 \Rightarrow x + y$ is divisible by 3
$\Rightarrow$ $x, y$ are multiples of 3 or one leaves remainder 1 and the other 2 when divided by 3.
3, 6, 9, ... , 30 are multiple of 3; 1, 4, 7, ..., 28 leave remainder 1; 2, 5, 8, ..., 29 leave remainder 2

$$\therefore \text{ Probability} = \frac{\binom{10}{1}\binom{10}{1} + \binom{10}{2}}{\binom{30}{2}} = \frac{145}{15 \times 29} = \frac{1}{3}$$

**50. (9) :** Let $E_1$ = the toss result is a head
$E_2$ = the toss result is a tail
$A$ = noted number is 7 or 8

$$\therefore \quad P(A) = P(E_1) \cdot P(A \mid E_1) + P(E_2) \cdot P(A \mid E_2)$$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{11} = \frac{193}{792} \quad \therefore \quad \frac{P}{792} = \frac{193}{792}$$

$$\therefore \quad P = 193$$

**51. (2) :** Divisors of $N = 2^{n-1}(2^n - 1)$ are
$1, 2, 2^2, ..., 2^{n-1}, 2^n - 1, 2(2^n - 1), 2^2(2^n - 1), ...$
$..., 2^{n-1}(2^n - 1)$

$$1 + \frac{1}{\alpha_1} + \frac{1}{\alpha_2} + ... + \frac{1}{\alpha_k} = 1 + \frac{1}{2} + .... + \frac{1}{2^{n-1}} + \frac{1}{2^n - 1} + \frac{1}{2(2^n - 1)} + ..... + \frac{1}{2^{n-1}(2^n - 1)}$$

$$= \left(1 + \frac{1}{2} + ... + \frac{1}{2^{n-1}}\right) + \frac{1}{2^n - 1}\left(1 + \frac{1}{2} + ... + \frac{1}{2^{n-1}}\right)$$

$$= \left(1 + \frac{1}{2} + ... + \frac{1}{2^{n-1}}\right)\left(1 + \frac{1}{2^n - 1}\right)$$

$$= \frac{1\left(1 - \left(\frac{1}{2}\right)^n\right)}{1 - \frac{1}{2}}\left(\frac{2^n}{2^n - 1}\right) = \frac{2^n}{2^{n-1}} = 2$$

**52. (4) :** Total ways of choosing $m$ and $n$ is $n(S)$
$= 100 \times 100$
Now, $7^1 = 7 = 5k + 2$, $7^2 = 49 = 5k + 4$,
$7^3 = 343 = 5k + 3$, $7^4 = 2401 = 5k + 1$
The same sequence will repeat for next four powers
$7^5 = 5k + 2$, $7^6 = 5k + 4$, $7^7 = 5k + 3$, $7^8 = 5k + 1$
$\therefore$ $7^1, 7^5, 7^9, ..., 7^{97}$ are of the type $5k + 2$
$7^2, 7^6, 7^{10}, ...., 7^{98}$ are of the type $5k + 4$
$7^3, 7^7, 7^{11}, ...., 7^{99}$ are of the type $5k + 3$
& $7^4, 7^8, 7^{12}, ..., 7^{100}$ are of the type $5k + 1$

There will be only four favourable combinations in which $7^m + 7^n$ will be divisible by 5.

$7^m : 5k + 2, 5k + 4, 5k + 3, 5k + 1$

$7^n : 5k + 3, 5k + 1, 5k + 2, 5k + 4$

$\therefore$ Number of favourable cases is

$n(E) = 25 \times 25 + 25 \times 25 + 25 \times 25 + 25 \times 25$

$= 4 \times 625$

$\therefore$ Required probability $= \frac{n(E)}{n(S)} = \frac{4 \times 625}{100 \times 100} = \frac{1}{4}$

**53. (3) :** Number of ways in which $m - 1$ remaining cars can take their places (excluding the car of a man)

$= {}^{n-1}C_{m-1}$

No. of ways in which remaining $(m - 1)$ cars can take places keeping the two places on two sides of his car vacant $= {}^{n-3}C_{m-1}$

Probability $= \frac{{}^{n-3}C_{m-1}}{{}^{n-1}C_{m-1}} = \frac{(n-m)(n-m-1)}{(n-1)(n-2)}$

$\Rightarrow A = 1, B = 2 \quad \Rightarrow \quad A + B = 3$

**54. (1) :** Let $E_1$, $E_2$, $E_3$ $E_4$, $E_5$ and $E_6$ be the events of occurrence of 1, 2, 3, 4, 5 and 6 on the dice respectively, and let $E$ be the event

$\therefore \; P(E_1) = \frac{1-k}{6}; \; P(E_2) = \frac{1+2k}{6}; \; P(E_3) = \frac{1-k}{6}$

$P(E_4) = \frac{1+k}{6}; \; P(E_5) = \frac{1-2k}{6}; P(E_6) = \frac{1+k}{6}$

and $\frac{1}{9} \le P(E) \le \frac{2}{9}$

Then, $E \equiv \{(3, 6), (6, 3), (4, 5), (5, 4)\}$

Hence,

$P(E) = P(E_3E_6) + P(E_6E_3) + P(E_4E_5) + P(E_5E_4)$

Since $E_1$, $E_2$, $E_3$, $E_4$, $E_5$ and $E_6$ are independent

$\therefore \; P(E) = P(E_3)P(E_6) + P(E_6)P(E_3) + P(E_4)P(E_5) + P(E_5)P(E_4)$

$= 2P(E_3)P(E_6) + 2P(E_4)P(E_5)$

$= 2\left(\frac{1-k}{6}\right)\left(\frac{1+k}{6}\right) + 2\left(\frac{1+k}{6}\right)\left(\frac{1-2k}{6}\right)$

$= \frac{1}{18}[2 - k - 3k^2]$

Since, $\frac{1}{9} \le P(E) \le \frac{2}{9} \quad \therefore \quad -\frac{1}{3} \le k \le 0$

$\therefore$ Set of integral value of $k = \{0\}$

$\therefore$ Number of integral solution of $k$ is 1

**55. (5) :** $n(S) = {}^{3n}C_2$

Let $A_0 = \{3, 6, 9, \ldots 3n\}$, $A_1 = \{1, 4, 7, \ldots, 3n - 2\}$

$A_2 = \{5, 8, 11, \ldots, 3n - 1\}$

$(x^2 - y^2)$ is divisible by 3. If both $x$, $y$ should come from $A_0$ or $A_1$ or $A_2$ or one is from $A_1$ and other from $A_2$.

$n(E) = 3\,{}^nC_2 + {}^nC_1 \times {}^nC_1 = \frac{n}{2}(5n - 3)$

$P(E) = \frac{5n-3}{3(3n-1)}$

**56. (8) :** $P(A) = \frac{{}^9C_2}{{}^{14}C_3} = \frac{9}{91}, \; P(B) = \frac{{}^{10}C_2}{{}^{14}C_3} = \frac{45}{364}$

$P(A \cap B) = \frac{{}^5C_1}{{}^{14}C_3} = \frac{5}{364}, \quad P(A \cup B) = \frac{19}{91}$

**57. (7) :** $P(G_k) \propto k^2 \Rightarrow P(G_k) = \lambda k^2$

$\sum_{k=0}^{n} P(G_k) = 1$ (as these are mutually exclusive and exhaustive events)

$\Rightarrow \; \lambda \sum_{k=0}^{n} k^2 = 1 \Rightarrow \; \lambda = \frac{6}{n(n+1)(2n+1)}$

$P(A) = \sum_{k=0}^{n} P(G_k)P(A/G_k)$

$= \sum_{k=0}^{n} \lambda k^2 \cdot \frac{k}{n} = \frac{\lambda}{n} \cdot \frac{n^2(n+1)^2}{4} = \frac{3(n+1)}{2(2n+1)}$

Take $n = 15$.


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# Math Archives 10 Best Problems

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

**1.** Let $\frac{d}{dx}(f(x)) = \frac{e^{\sin x}}{x}, x > 0.$

If $\int_1^4 \frac{2e^{\sin x^2}}{x} dx = f(k) - f(1)$, then one of the possible value of $k$ is

(a) $-4$ (b) 0 (c) 2 (d) 16

**2.** $\int \sqrt{x-3}\,(\sin^{-1}(\ln x) + \cos^{-1}(\ln x))\,dx$, is equal to

(a) $\frac{\pi}{3}(x-3)^{3/2} + c$ (b) 0

(c) does not exist (d) none of these

**3.** $\int \frac{dx}{(x-1)\sqrt{x^2-1}}$ equals

(a) $-\sqrt{\frac{x-1}{x+1}} + c$ (b) $\sqrt{\frac{x-1}{x+1}} + c$

(c) $\sqrt{\frac{x+1}{x-1}} + c$ (d) $-\sqrt{\frac{x+1}{x-1}} + c$

**4.** The degree of the differential equation of the family of lines given by $y = cx + \sqrt{c}$ is

(a) 1 (b) 2

(c) 4 (d) not defined

**5.** The order of the differential equation of the family of rectangular hyperbolas having fixed centre, is

(a) 1 (b) 2 (c) 3 (d) 4

**6.** Let $f$ be a differentiable function such that $f(x) = x^2 + \int_0^x e^{-t} f(x-t)\,dt$, then prove that

$f(x) = \frac{x^3}{3} + x^2.$

**7.** If $f : R - \{0\} \to R$ is a differentiable function such that $\int_1^{xy} f(t)\,dt = y\int_1^x f(t)\,dt + x\int_1^y f(t)\,dt, \forall\, x,\ y \in R - \{0\}$, $f(1) = 1$, then find $f(x)$.

**8.** Evaluate :

$$I = \int \frac{(\sin\theta - \cos\theta)\,d\theta}{(\cos\theta + \sin\theta)\sqrt{\sin\theta\cdot\cos\theta + \sin^2\theta\cdot\cos^2\theta}}.$$

**9.** If $f$ is successively differentiable function, then prove that

$$f(x) = f(0) + xf'(0) + \frac{1}{2!}x^2 f''(0) + \frac{1}{3!}x^3 f'''(0) + \frac{1}{3!}\int_0^x f''''(t)(x-t)^3\,dt$$

**10.** Prove that in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the distance between the center and any normal does not exceed $|a - b|$. Find a point on the ellipse normal at which is at a distance of $|a - b|$ from centre.

## SOLUTIONS

**1. (d) :** Given $\frac{d}{dx}(f(x)) = \frac{e^{\sin x}}{x}, x > 0$

Let $I = \int_1^4 \frac{2e^{\sin x^2}}{x} dx$

Put $z = x^2 \Rightarrow dz = 2xdx$

$\therefore\ I = \int_1^4 \frac{2xe^{\sin x^2}}{x^2} dx = \int_1^{16} \frac{e^{\sin z}}{z} dz$

$= [f(z)]_1^{16} = f(16) - f(1) \Rightarrow f(k) = f(16)$

$\therefore$ One possible value of $k$ is 16.

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**2. (c) :** For $x > e$, $\ln x > 1$

$\Rightarrow$ $\sin^{-1}(\ln x) + \cos^{-1}(\ln x)$ is not defined.

Hence, the given integral does not exist.

**3. (d) :** Let $I = \int \dfrac{dx}{(x-1)^2 \cdot \left(\dfrac{x+1}{x-1}\right)^{1/2}}$

Put $u = \dfrac{x+1}{x-1} \Rightarrow -\dfrac{1}{2}du = \left(\dfrac{1}{x-1}\right)^2 dx$

$\therefore \quad I = -\dfrac{1}{2}\int \dfrac{du}{\sqrt{u}} = -\sqrt{u} + c = -\sqrt{\dfrac{x+1}{x-1}} + c$

**4. (b) :** We have, $y = cx + \sqrt{c} \Rightarrow \dfrac{dy}{dx} = c$

$\Rightarrow y = x\dfrac{dy}{dx} + \sqrt{\dfrac{dy}{dx}} \Rightarrow \dfrac{dy}{dx} = \left(y - x\dfrac{dy}{dx}\right)^2$

$\Rightarrow$ degree is 2.

**5. (b) :** We need one parameter to fix the direction of the axes and one parameter for fixing the length of transverse axis of the hyperbola (note that eccentricity is $\sqrt{2}$ ). Hence, required order is 2.

**6.** Given, $f(x) = x^2 + \int_0^x e^{-t} f(x-t)\,dt$

$\therefore \quad f(x) = x^2 + \int_0^x e^{-(x-t)} f(t)\,dt$

$$\left(\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right)$$

or $\quad f(x) = x^2 + e^{-x}\int_0^x e^t f(t)\,dt \qquad \ldots (1)$

Differentiating (1) w.r.t. $x$, we get

$$f'(x) = 2x + e^{-x}[1 \cdot e^x f(x) - 0] + e^{-x}(-1)\int_0^x e^t f(t)\,dt$$

$$\therefore \quad f'(x) = 2x + f(x) - e^{-x}\int_0^x e^t f(t)\,dt \qquad \ldots (2)$$

Adding (1) and (2), we get

$f'(x) + f(x) = x^2 + 2x + f(x)$

$\therefore \quad f'(x) = x^2 + 2x \qquad \ldots (3)$

Integrating, we get $f(x) = \dfrac{x^3}{3} + x^2 + C$

$$\therefore \quad f(x) = \frac{x^3}{3} + x^2 + C \qquad \ldots (4)$$

Now, putting $x = 0$ in (1), we get $f(0) = 0$

Then put $x = 0$ in (4), we get $f(0) = C \Rightarrow C = 0$.

$$\therefore \quad f(x) = \frac{x^3}{3} + x^2$$

**7.** Differentiating partially w.r.t. $y$, we get

$$f(xy)\cdot x = \int_1^x f(t)\,dt + xf(y)$$

Putting $y = 1$, we get

$$xf(x) = \int_1^x f(t)\,dt + xf(1) = \int_1^x f(t)\,dt + x, \text{ as } f(1) = 1.$$

Differentiating w.r.t. $x$, we get

$f(x) + xf'(x) = f(x) + 1$

$$\Rightarrow \quad f'(x) = \frac{1}{x} \quad \Rightarrow \quad f(x) = \ln|x| + C$$

Now $f(1) = 1 \Rightarrow C = 1 \Rightarrow f(x) = 1 + \ln|x|$.

**8.** On multiplying the numerator and denominator by $(\sin\theta + \cos\theta)$, we get

$$I = \int \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)\,d\theta}{(\cos\theta + \sin\theta)^2 \sqrt{\sin\theta \cdot \cos\theta + \sin^2\theta \cdot \cos^2\theta}}$$

$$\therefore \quad I = \int \frac{-(\cos^2\theta - \sin^2\theta)\,d\theta}{(1+\sin 2\theta)\sqrt{\dfrac{\sin 2\theta}{2} + \dfrac{\sin^2 2\theta}{4}}}$$

$$= \int \frac{-\cos 2\theta\, d\theta}{(1+\sin 2\theta)\sqrt{\dfrac{1}{2}\sin 2\theta + \dfrac{1}{4}\sin^2 2\theta}}$$

Put $1 + \sin 2\theta = x \Rightarrow 2\cos 2\theta\, d\theta = dx$

$$\therefore \quad I = \int \frac{-\dfrac{dx}{2}}{x \cdot \sqrt{\dfrac{(x-1)}{2} + \dfrac{(x-1)^2}{4}}}$$

$$= \int \frac{-dx}{x\sqrt{2(x-1) + (x^2 - 2x + 1)}} = \int \frac{-dx}{x\sqrt{x^2 - 1}}$$

$= \text{cosec}^{-1}x + c$

$= \text{cosec}^{-1}(1 + \sin 2\theta) + c$

**9.** $\int_0^x f''''(t)(x-t)^3\,dt = \left[(x-t)^3 f'''(t)\right]_0^x$

$$+3\int_0^x (x-t)^2 f'''(t)\,dt$$

$$= -x^3 f'''(0) + 3\left[(x-t)^2 f''(t)\Big|_0^x + 2\int_0^x (x-t) f''(t)\,dt\right]$$

$$= -x^3 f'''(0) + 3[-x^2 f''(0)] + 6\int_0^x (x-t) f''(t)\, dt$$

$$= -x^3 f'''(0) - 3x^2 f''(0) + 6\left[ (x-t) f'(t)\Big|_0^x + \int_0^x f'(t)\, dt \right]$$

$$= -x^3 f'''(0) - 3x^2 f''(0) + 6[-x f'(0)] + 6\int_0^x f'(t)\, dt$$

$$= -x^3 f'''(0) - 3x^2 f''(0) - 6xf'(0) + 6\big[f(t)\big]_0^x$$

$$\Rightarrow \int_0^x f''''(t)(x-t)^3\, dt = -x^3 f'''(0) - 3x^2 f''(0) - 6xf'(0) + 6\big[f(x) - f(0)\big]$$

$$\therefore \quad f(x) - f(0) = \frac{1}{6}\Big[\int_0^x f''''(t)(x-t)^3\, dt + x^3 f'''(0) + 3x^2 f''(0) + 6x f'(0)\Big]$$

$$\therefore\ f(x) = f(0) + xf'(0) + \frac{1}{2!}x^2 f''(0) + \frac{1}{3!}x^3 f'''(0) + \frac{1}{3!}\int_0^x f''''(t)(x-t)^3\, dt$$

**10.** Let $P(a\cos\theta, b\sin\theta)$ represents any point on the given ellipse and the slope of the tangent at this point is given by

$$\frac{2x}{a^2} + \left(\frac{2y}{b^2}\right)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{b^2 x}{a^2 y} = -\frac{b\cot\theta}{a}, \text{ at } P.$$

Therefore, the slope of the normal at the point $P$ is $(a\tan\theta)/b$, so that the equation of the normal is

$$y - b\sin\theta = \frac{a\tan\theta}{b}(x - a\cos\theta)$$

$$\Rightarrow \left(-\frac{a\tan\theta}{b}\right)x + y + \left(-b\sin\theta + \frac{a^2\sin\theta}{b}\right) = 0 \quad \ldots(1)$$

The distance from (0, 0) to the line (1) is

$$D = \left|\frac{b\sin\theta - \frac{a^2\sin\theta}{b}}{\sqrt{\frac{a^2\tan^2\theta}{b^2} + 1^2}}\right| = \left|\frac{(b^2 - a^2)\sin\theta}{\sqrt{b^2 + a^2\tan^2\theta}}\right|$$

$$= \left|\frac{b^2 - a^2}{\sqrt{b^2\operatorname{cosec}^2\theta + a^2\sec^2\theta}}\right|$$

For fixed $a$ and $b$, $D$ is maximum when $S = b^2\operatorname{cosec}^2\theta + a^2\sec^2\theta$ is minimum. This happens when

$$\frac{dS}{d\theta} = -2b^2\operatorname{cosec}\theta(\operatorname{cosec}\theta\cot\theta) + 2a^2\sec\theta(\sec\theta\tan\theta)$$

$$= -\frac{2b^2\cos\theta}{\sin^3\theta} + \frac{2a^2\sin\theta}{\cos^3\theta} = -\frac{2(b^2\cos^4\theta - a^2\sin^4\theta)}{\sin^3\theta\cos^3\theta} = 0$$

*i.e.*, when $\tan^2\theta = b/a$.

Differentiating $\frac{dS}{d\theta}$, we get

$$\frac{d^2S}{d\theta^2} = \frac{2b^2\sin\theta}{\sin^3\theta} + \frac{6b^2\cos^2\theta}{\sin^4\theta} + \frac{2a^2\cos\theta}{\cos^3\theta} + \frac{6a^2\sin^2\theta}{\cos^4\theta}$$

$$= \frac{2b^2}{\sin^2\theta} + \frac{6b^2\cos^2\theta}{\sin^4\theta} + \frac{2a^2}{\cos^2\theta} + \frac{6a^2\sin^2\theta}{\cos^4\theta} > 0,\ \forall\theta$$

Therefore $\tan^2\theta = b/a$ corresponds to the minimum value of $S$ given by

$$S_{\min} = b^2(1 + \cot^2\theta) + a^2(1 + \tan^2\theta)\Big|_{\tan^2\theta = \frac{b}{a}}$$

$$= b^2\left(1 + \frac{a}{b}\right) + a^2\left(1 + \frac{b}{a}\right) = (a+b)^2$$

Hence, the maximum value of the required distance is

$$D_{\max} = \left|\frac{b^2 - a^2}{a+b}\right| = |b - a|$$

Hence $D \le |b - a|$.

Equality occurs when $\tan^2\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\frac{\sqrt{b}}{\sqrt{a}}$

$$\Rightarrow \sin\theta = \sqrt{\frac{b}{a+b}}, \cos\theta = \sqrt{\frac{a}{a+b}}$$

$$\Rightarrow P \equiv \left(a\sqrt{\frac{a}{a+b}}, b\sqrt{\frac{b}{a+b}}\right)$$

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# ACE YOUR WAY CBSE XII

## VECTOR ALGEBRA AND 3D-GEOMETRY

## VECTOR ALGEBRA

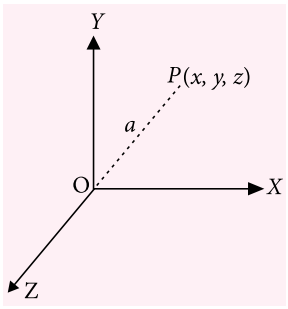
### VECTOR

- Vector quantities are those which have both magnitude and direction.
- A vector, denoted by $\overrightarrow{PQ}$, is determined by two points $P$, $Q$ such that the magnitude of the vector is the length of the straight line $PQ$ and its direction is that from $P$ to $Q$. The point $P$ is called the initial point and $Q$ is called the terminal point of vector $\overrightarrow{PQ}$.
- The magnitude of a vector $\vec{a}$ is the positive number which is the measure of its length and is denoted by $|\vec{a}|$.

#### Position Vector

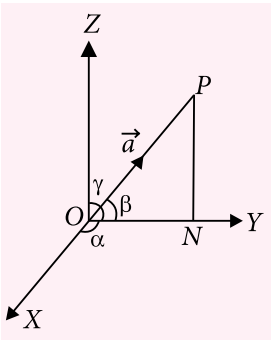
- If $P$ be a point in a space, having co-ordinates $(x, y, z)$ with respect to the origin $O(0, 0, 0)$. Then the magnitude of $\overrightarrow{OP}$ is given by

$$|\overrightarrow{OP}| = \sqrt{x^2 + y^2 + z^2}.$$




### DIRECTION COSINES

- If a vector $\vec{a}$ makes angles, $\alpha$, $\beta$, $\gamma$ with the positive direction of $X$, $Y$ and $Z$ axes respectively, then $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are called direction cosines of vector $\vec{a}$ and are usually denoted by $l$, $m$, $n$.




### DIRECTION RATIOS

- Any three numbers $a$, $b$, $c$ proportional to the direction cosines $l$, $m$, $n$ respectively of a line, are called direction ratios of the line.

$$\therefore \quad \frac{l}{a} = \frac{m}{b} = \frac{n}{c}$$

***Note* :**

(i) The direction cosines of a line are

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}},$$

$$n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}},$$

where $a$, $b$, $c$ are direction ratios.

(ii) $l^2 + m^2 + n^2 = 1$

### TYPES OF VECTORS

**(i) Zero or Null vector :** A vector whose initial and terminal points are coincident is called the zero or the null vector.
The null vector is denoted by $\vec{0}$.

**(ii) Unit Vector :** A vector whose modulus is unity, is called a unit vector. The unit vector in the direction of a vector $\vec{a}$ is denoted by $\hat{a}$, read as '$a$ cap'. Thus, $|\hat{a}| = 1$.

**(iii) Like and Unlike Vectors :** Vectors are said to be like when they have the same sense of direction and unlike when they have opposite direction.

**(iv) Collinear or Parallel Vectors :** Vectors having the same or parallel supports are called collinear vectors.

**(v) Co-initial Vectors :** Vectors having the same initial point are called co-initial vectors.

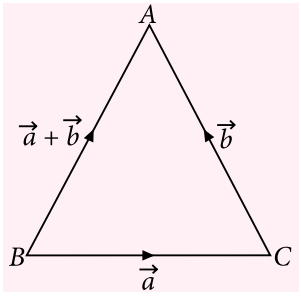
**(vi) Equal vectors:** Two vectors $\vec{a}$ and $\vec{b}$ are said to be equal, if they have the same magnitude and direction irrespective of the positions of their initial points and written as $\vec{a} = \vec{b}$.

**(vii) Negative of a Vector :** The vector which has the same magnitude as the vector $\vec{a}$ but opposite direction is called the negative of $\vec{a}$ and is denoted by $-\vec{a}$. Thus, if $\overrightarrow{PQ} = \vec{a}$, then $\overrightarrow{QP} = -\vec{a}$.

**(viii) Free Vectors :** Vectors whose initial point is not specified are called free vectors.

## ADDITION OF VECTORS

- **Triangle law:** If two vectors are represented in magnitude and direction by the two sides of a triangle taken in order, then their sum can be represented in magnitude and direction by the third side of the triangle taken in reverse order.



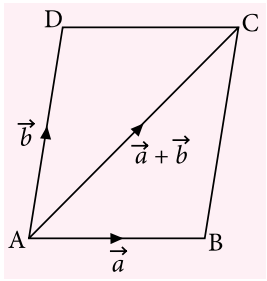

Thus in the given figure,

$\overrightarrow{BC}+\overrightarrow{CA}=\overrightarrow{BA}$

$\Rightarrow \vec{a}+\vec{b}=\vec{a}+\vec{b}$

Similarly $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$, $\overrightarrow{CA}+\overrightarrow{AB}=\overrightarrow{CB}$

- **Parallelogram law of addition :** If two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram, then their sum or resultant will be represented in magnitude and direction by the diagonal of the parallelogram passing through the point of intersection of the two adjacent sides.




Thus, if $\vec{a}$ and $\vec{b}$ are represented by $\overrightarrow{AB}$ and $\overrightarrow{AD}$ respectively, then $\vec{a}+\vec{b}$ will be represented by $\overrightarrow{AC}$.

### Multiplication of a Vector by a Scalar

Let $\vec{a}$ be any vector and $m$ be any scalar. Then the product $m\vec{a}$ is defined as a vector whose magnitude is $|m|$ times that of $\vec{a}$ and the direction is (i) same as that of $\vec{a}$ if $m$ is positive and (ii) opposite to that of $\vec{a}$ if $m$ is negative.

### Properties of Vector Addition

For any three vectors, $\vec{a}, \vec{b}, \vec{c}$, we have the following properties :

| | |
|---|---|
| Commutative law | $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ |
| Associative law | $\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}$ |
| Additive identity | $\vec{a}+\vec{0}=\vec{a}=\vec{0}+\vec{a}$ |
| Additive inverse | $\vec{a}+(-\vec{a})=\vec{0}$ |
| Multiplication by scalar | $m(\vec{a})=(\vec{a})m=m\vec{a}$,<br>$m(n\vec{a}) = n(m\vec{a}) = (mn)\vec{a}$,<br>$(m+n)\vec{a}=m\vec{a} + n\vec{a}$,<br>$m(\vec{a}+\vec{b})=m\vec{a}+m\vec{b}$,<br>where $m$ and $n$ are any two scalars. |

## COMPONENTS OF A VECTOR

- If $P(x, y, z)$ is a point in space and $\hat{i},\hat{j},\hat{k}$ are unit vectors along $X$, $Y$ and $Z$-axes respectively, then the vector components of $\overrightarrow{OP}$ along $X$, $Y$ and $Z$-axes respectively are $x\hat{i}$, $y\hat{j}$ and $z\hat{k}$. Also, $x$, $y$ and $z$ are called scalar components of $\overrightarrow{OP}$.

$$\overrightarrow{OP}=x\hat{i}+y\hat{j}+z\hat{k}$$

$$\Rightarrow \quad |\overrightarrow{OP}|=\sqrt{x^2+y^2+z^2}$$

For any two vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, we define

(a) $\vec{a}+\vec{b}=(a_1+b_1)\hat{i}+(a_2+b_2)\hat{j}+(a_3+b_3)\hat{k}$

(b) $\vec{a}-\vec{b}=(a_1-b_1)\hat{i}+(a_2-b_2)\hat{j}+(a_3-b_3)\hat{k}$

(c) $m\vec{a}=(ma_1)\hat{i}+(ma_2)\hat{j}+(ma_3)\hat{k}$, where $m$ is a scalar

(d) $\vec{a}=\vec{b}\Leftrightarrow a_1=b_1, a_2=b_2$ and $a_3 = b_3$

## VECTOR JOINING TWO POINTS

If $P\equiv(x_1, y_1, z_1)$ and $Q\equiv(x_2, y_2, z_2)$ be any two points, then the vector joining $P$ and $Q$ is given by

$$\overrightarrow{PQ}=\overrightarrow{OQ}-\overrightarrow{OP}=(x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k}$$

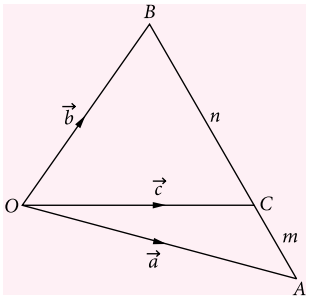
Also, $PQ=|\overrightarrow{PQ}|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

## SECTION FORMULA

### Internal Division

- Let $A$ and $B$ be any two points with position vectors $\vec{a}$ and $\vec{b}$ respectively, and let $C$ be a point dividing $AB$ inter-nally in the ratio $m:n$. Then the position vector of $C$ is given by

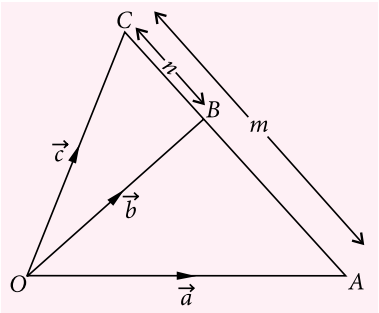
$$\overrightarrow{OC}=\frac{m\vec{b}+n\vec{a}}{m+n}$$



### External Division

- Let $A$ and $B$ be any two points with position vectors $\vec{a}$ and $\vec{b}$ respectively and let $C$ be a point dividing $AB$ externally in the ratio $m : n$. Then, the position vector of $C$ is given by

$$\overrightarrow{OC} = \frac{m\vec{b} - n\vec{a}}{m-n}$$




### Mid Point formula

- If $C$ is the mid point of $AB$, then it divides $AB$ in the ratio 1 : 1. So, position vector of $C$ is given by

$$\frac{1\cdot\vec{a}+1\cdot\vec{b}}{1+1} = \frac{\vec{a}+\vec{b}}{2}$$

## DOT OR SCALAR PRODUCT OF TWO VECTORS

Dot or scalar product of two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle $\theta$ is $ab\cos\theta$. It is denoted by $\vec{a}\cdot\vec{b}$

Thus, $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = ab\cos\theta$

**Properties of Scalar Product**

**(i)** $\vec{a}\cdot\vec{b}$ is a real number.

**(ii)** $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ (Commutative law)

**(iii)** $(x\vec{a})\cdot\vec{b} = x(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(x\vec{b})$, where $x$ is a scalar (Associative law)

**(iv)** $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$ (Distributive law)

**(v)** $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

**(vi)** If $\vec{a}$ is perpendicular to $\vec{b}$, then $\vec{a}\cdot\vec{b} = 0$

**(vii)** $\vec{a}\cdot\vec{b} < 0$, if and only if the angle between $\vec{a}$ and $\vec{b}$ is obtuse.

**(viii)** $\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a} \Rightarrow 0$ or $\vec{b} = 0$ or $\vec{a} \perp \vec{b}$.

**(ix)** $\hat{i}\cdot\hat{i} = 1, \hat{j}\cdot\hat{j} = 1, \hat{k}\cdot\hat{k} = 1, \hat{i}\cdot\hat{j} = 0, \hat{j}\cdot\hat{k} = 0, \hat{i}\cdot\hat{k} = 0$

**(x)** If the two vectors have same direction, then $\theta = 0 \therefore \cos\theta = 1 \quad \therefore \vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$

**(xi)** If the two vectors have opposite direction, then $\theta = \pi$

$\therefore \cos\theta = -1 \therefore \vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$

**(xii)** If $\hat{a}$ and $\hat{b}$ are unit vectors, then $|\hat{a}| = 1, |\hat{b}| = 1 \therefore \hat{a}\cdot\hat{b} = 1\cdot1\cdot\cos\theta = \cos\theta$

**(xiii)** If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Then, $\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

**(xiv)** If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\theta$ be the angle between two vectors $\vec{a}$ and $\vec{b}$ then

$$\theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{ab}\right)$$

$$\therefore \theta = \cos^{-1}\left(\frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}\sqrt{b_1^2 + b_2^2 + b_3^2}}\right)$$

## PROJECTION OF A VECTOR ON A LINE

- Projection of a vector $\vec{a}$ on the other vector $\vec{b}$, is given by

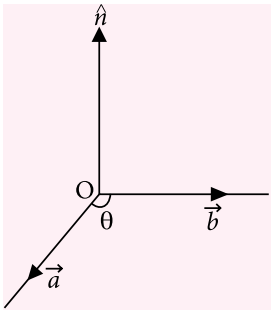
$$\vec{a}\cdot\hat{b} \text{ or } \vec{a}\cdot\left(\frac{\vec{b}}{|\vec{b}|}\right) \text{ or } \frac{1}{|\vec{b}|}(\vec{a}\cdot\vec{b})$$

- If $\theta = 0$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{AB}$ itself and if $\theta = \pi$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{BA}$.

- If $\theta = \dfrac{\pi}{2}$ or $\theta = \dfrac{3\pi}{2}$, then the projection vector of $\overrightarrow{AB}$ will be zero vector.

## VECTOR OR CROSS PRODUCT OF TWO VECTORS

- The vector product of two vectors $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\times\vec{b}$ and read as $\vec{a}$ cross $\vec{b}$. Vector product is also called cross product. Thus, $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$, where $\hat{n}$ is the unit vector along $\vec{a}\times\vec{b}$.




### Properties of Cross Product

**(i)** $\vec{a}\times\vec{b}$ is a vector.

**(ii)** If $\vec{a}$ and $\vec{b}$ be any two non zero vectors, then $\vec{a}\times\vec{b} = \vec{0} \Leftrightarrow \vec{a}\,||\,\vec{b}$ or angle between $\vec{a}$ and $\vec{b}$ *i.e.*, $\theta = 0$ or $\pi$.

**(iii)** If $\theta = \dfrac{\pi}{2}$ then $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|$.

**(iv)** For mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, we have

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$

$\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$

**(v)** The angle between two vectors $\vec{a}$ and $\vec{b}$ is given as $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$

**(vi)** The vector product is not commutative, as $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$.

$\Rightarrow \hat{j}\times\hat{i} = -\hat{k},\ \hat{k}\times\hat{j} = -\hat{i}$ and $\hat{i}\times\hat{k} = -\hat{j}$

**(vii)** If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle then its area is given by $\frac{1}{2}|\vec{a}\times\vec{b}|$.

**(viii)** If $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a parallelogram, then its area is given by $|\vec{a}\times\vec{b}|$.

**(ix)** If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three vectors and $\lambda$ be a scalar, then

(a) $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$

[Distributivity of vector product over addition]

(b) $\lambda(\vec{a}\times\vec{b}) = (\lambda\vec{a})\times\vec{b} = \vec{a}\times(\lambda\vec{b})$

**(x)** If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ *and* $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$,

then $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

## 3D-GEOMETRY

### DIRECTION COSINES OF A LINE PASSING THROUGH TWO POINTS

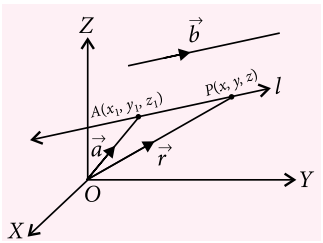
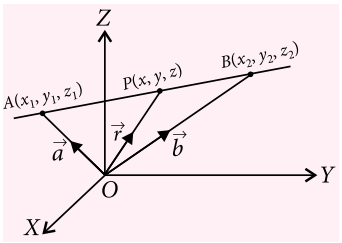
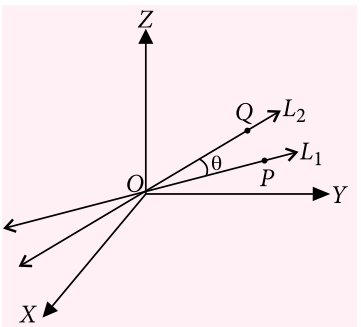
The direction ratios of the line segment joining $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z)$, is

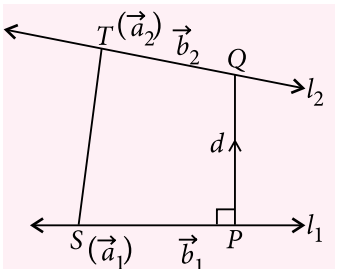
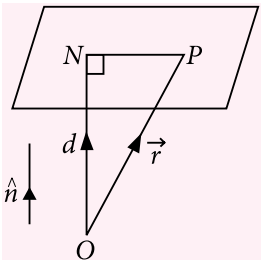
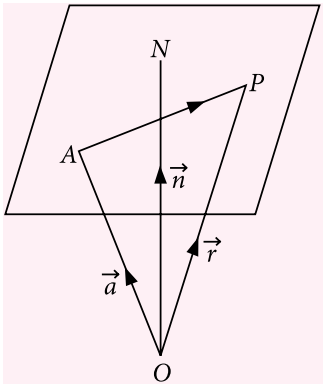
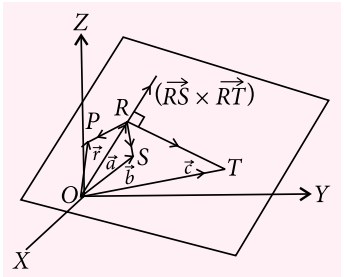
$x_2 - x_1, y_2 - y_1, z_2 - z_1$ or $x_1 - x_2, y_1 - y_2, z_1 - z_2$

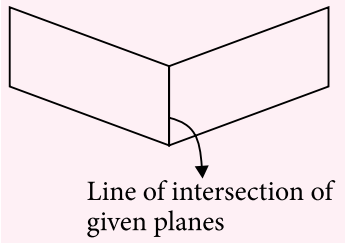
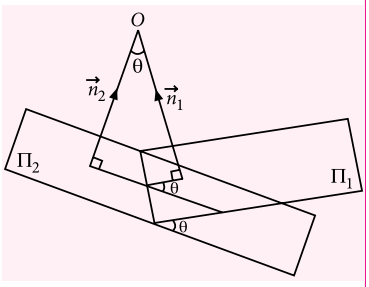
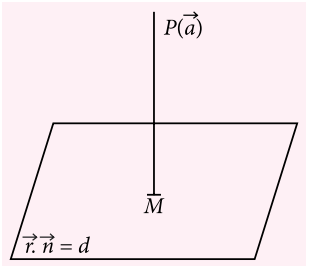
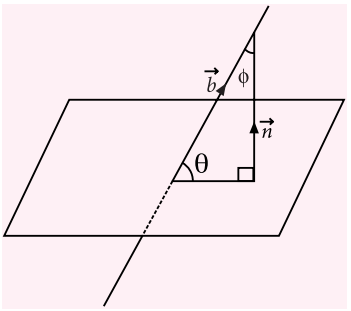
So, the direction cosines of the line segment joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are

$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$, where

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

| Equation | Vector Form | Cartesian form | Figure |
|---|---|---|---|
| Equation of a line passing through a given point and parallel to a given vector $\vec{b}$ | $\vec{r} = \vec{a} + \lambda\vec{b}$ | $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$. If the direction cosines $<l, m, n>$ is given, then equation of the line is $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ |   |
| Equation of a line passing through two given points | $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}),\ \lambda \in R$ | $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$ |   |
| Angle between two lines | Let the vector equations of two lines $L_1$ and $L_2$ be $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ Then, $\cos\theta = \left\|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right\|$ | Let the cartesian equations of two lines be $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ Then, $\cos\theta = \left\|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right\|$ |   |

| | | | |
|---|---|---|---|
| Shortest distance between two lines<br>• Distance between two skew lines | Let $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ and $\vec{r}=\vec{a}_2+\lambda\vec{b}_2$ represents two skew lines, then the magnitude of the shortest distance ($d$) is $\left\|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{\|\vec{b}_1\times\vec{b}_2\|}\right\|$ | The shortest distance ($d$) between the lines, $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ is $d=\left\|\frac{\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}}\right\|$ |  |
| • Distance between two parallel lines | $\left\|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{\|\vec{b}\|}\right\|$ | | |
| • Condition for Coplanarity of two lines | $(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)=0$ | $\begin{vmatrix}x_2-x_1 & y_2-y_1 & z_2-z_1\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0$ | |
| Equation of a plane in<br>• Normal form | $\vec{r}\cdot\hat{n}=d$ | $lx+my+nz=d$ |  |
| • Intercept form | | $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ | |
| Equation of a plane perpendicular to a given vector and passing through a given point | Let $\vec{r}$ be the position vector of any point $P(x, y, z)$ in the plane, then $(\vec{r}-\vec{a})\cdot\vec{n}=0$ | $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$ |  |
| Equation of a plane passing through three non-collinear points | $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$ | $\begin{vmatrix}x-x_1 & y-y_1 & z-z_1\\ x_2-x_1 & y_2-y_1 & z_2-z_1\\ x_3-x_1 & y_3-y_1 & z_3-z_1\end{vmatrix}=0$ |  |

| | | | |
|---|---|---|---|
| Equation of the plane passing through the intersection of two given planes | $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ | $(a_1x+b_1y+c_1z-d_1)+\lambda(a_2x+b_2y+c_2z-d_2)=0$ |   |
| Angle between two planes | $\cos\theta=\left\|\frac{\vec{n}_1\cdot\vec{n}_2}{\|\vec{n}_1\|\|\vec{n}_2\|}\right\|$ | $\cos\theta=\left\|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right\|$ planes to be (i) perpendicular if $\cos\theta=a_1a_2+b_1b_2+c_1c_2=0$ (ii) parallel if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ |   |
| Distance of a point from a plane | $\frac{\|\vec{a}\cdot\vec{n}-d\|}{\|\vec{n}\|}$, where $\vec{n}$ is normal to the plane. | $\left\|\frac{ax_1+by_1+cz_1-d}{\sqrt{a^2+b^2+c^2}}\right\|$ |   |
| Angle between a line and a plane | $\theta=\sin^{-1}\left\|\frac{\vec{b}\cdot\vec{n}}{\|\vec{b}\|\|\vec{n}\|}\right\|$ | |   |

## Very Short Answer Type

1. Find the values of $x$ and $y$ so that the vectors $2\hat{i}+3\hat{j}$ and $x\hat{i}+y\hat{j}$ are equal.
2. Find the scalar and vector components of the vector with initial point $A(2, 1)$ and terminal point $B(-5, 7)$.
3. Find the value of $p$ for which the vectors $\vec{a}=3\hat{i}+2\hat{j}+9\hat{k}$ and $\vec{b}=\hat{i}+p\hat{j}+3\hat{k}$ are perpendicular.
4. Find the equation of a line passing through a point $(2, -1, 3)$ and parallel to the line $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}+\hat{j}-2\hat{k})$.
5. Find the direction cosines of the line segment joining the points $A(7, -5, 9)$ and $B(5, -3, 8)$.

## Short Answer Type

6. Let $\vec{a}=2\hat{i}+\hat{k}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$ be three vectors. Find a vector $\vec{r}$ which satisfies $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a}=0$.
7. The projection of a vector on the coordinate axes are 6, –3, 2. Find its length and direction cosines.
8. Find the distance of the point $(2\hat{i}-\hat{j}-4\hat{k})$ from the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=9$.

**9.** Find a unit vector perpendicular to the plane $ABC$ where $A$, $B$, $C$ are the points (3, –1, 2), (1, –1, –3), (4, –3, 1) respectively.

**10.** Find the equation of a plane through the intersection of the planes $\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})=5$ and $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$ and passing through the point (2, 1, –2).

**11.** Find the direction cosines of the perpendicular from the origin to the plane $\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})+3=0$.

## Long Answer Type

**12.** (i) Find the vector equation of the plane passing through the points $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$.

(ii) Find the angle between the line $\vec{r}=(\hat{i}+2\hat{j}-\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=4$.

**13.** Find the shortest distance and the vector equation of the line of shortest distance between the lines given by $\vec{r}=3\hat{i}+8\hat{j}+3\hat{k}+\lambda(3\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=-3\hat{i}-7\hat{j}+6\hat{k}+\mu(-3\hat{i}+2\hat{j}+4\hat{k})$

**14.** (i) The vertices of a triangle are $A(5, 4, 6)$, $B(1, -1, 3)$ and $C(4, 3, 2)$. The internal bisector of $\angle BAC$ meets $BC$ in $D$. Find $AD$.

(ii) Find a vector of magnitude 15, which is perpendicular to both the vectors $4\hat{i}-\hat{j}+8\hat{k}$ and $-\hat{j}+\hat{k}$.

**15.** Find the image of the point having position vector $\hat{i}+3\hat{j}+4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$.

**16.** Show that the line joining the points (6, –4, 4) and (0, 0, –4) intersects the line joining the points (–1, –2, –3) and (1, 2, –5).

## SOLUTIONS

**1.** We know that,

$a_1\hat{i}+b_1\hat{j}=a_2\hat{i}+b_2\hat{j}$

$\Leftrightarrow$ $a_1=a_2$ and $b_1=b_2$

$\therefore$ $2\hat{i}+3\hat{j}=x\hat{i}+y\hat{j} \Rightarrow x=2, y=3$

**2.** Since $\overrightarrow{AB}=(-5\hat{i}+7\hat{j})-(2\hat{i}+1\hat{j})$

$\Rightarrow$ $\overrightarrow{AB}=-7\hat{i}+6\hat{j}$

So, scalar components of $\overrightarrow{AB}$ along $OX$ and $OY$ are –7 and 6 respectively. The vector components of $\overrightarrow{AB}$ are $-7\hat{i}$ and $6\hat{j}$ along $OX$ and $OY$ respectively.

**3.** If vectors $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\cdot\vec{b}=0$

$\Rightarrow (3\hat{i}+2\hat{j}+9\hat{k})\cdot(\hat{i}+p\hat{j}+3\hat{k})=0$

$\Rightarrow 3+2p+27=0 \Rightarrow p=-15$

**4.** The given line is parallel to the vector $2\hat{i}+\hat{j}-2\hat{k}$ and the required line is parallel to the given line. So, required line is parallel to the vector $2\hat{i}+\hat{j}-2\hat{k}$. Thus, the equation of the required line passing through (2, –1, 3) is

$\vec{r}=(2\hat{i}-\hat{j}+3\hat{k})+\mu(2\hat{i}+\hat{j}-2\hat{k})$

**5.** Direction ratios of the line segment $AB$ are (5 – 7), –3 – (–5), (8 –9), *i.e.*, –2, 2, –1.

$\therefore$ its d.c.'s are

$$\frac{-2}{\sqrt{(-2)^2+2^2+(-1)^2}}, \frac{2}{\sqrt{(-2)^2+2^2+(-1)^2}}, \frac{-1}{\sqrt{(-2)^2+2^2+(-1)^2}}$$

*i.e.*, $\frac{-2}{3}, \frac{2}{3}, \frac{-1}{3}$.

**6.** We have,

$\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

$\Rightarrow$ $\vec{r}\times\vec{b}-\vec{c}\times\vec{b}=\vec{0}$

$\Rightarrow$ $(\vec{r}-\vec{c})\times\vec{b}=\vec{0}$

$\Rightarrow$ $\vec{r}-\vec{c}$ is parallel to $\vec{b}$

$\Rightarrow$ $\vec{r}-\vec{c}=\lambda\vec{b}$ for some scalar $\lambda$.

$\Rightarrow$ $\vec{r}=\vec{c}+\lambda\vec{b}$

Now, $\vec{r}\cdot\vec{a}=0$

$\Rightarrow$ $(\vec{c}+\lambda\vec{b})\cdot\vec{a}=0$

$\Rightarrow$ $(\vec{c}\cdot\vec{a})+\lambda(\vec{b}\cdot\vec{a})=0$

$\Rightarrow$ $\lambda=-\left(\frac{\vec{c}\cdot\vec{a}}{\vec{b}\cdot\vec{a}}\right)$

$\Rightarrow$ $\lambda=-\left(\frac{8+7}{2+0+1}\right)=-\frac{15}{3}=-5$

$\therefore$ $\vec{r}=\vec{c}-5\vec{b}=(4\hat{i}-3\hat{j}+7\hat{k})$

$-5(\hat{i}+\hat{j}+\hat{k})=-\hat{i}-8\hat{j}+2\hat{k}$

**7.** Let $l, m, n$ be the direction cosines of the given vector $\vec{r}$(say). Then, its projections on the coordinate axes are $l|\vec{r}|, m|\vec{r}|, n|\vec{r}|$.

$\therefore$ $l|\vec{r}|=6, m|\vec{r}|=-3, n|\vec{r}|=2$ ...(i)

$\Rightarrow$ $\{l|\vec{r}|\}^2+\{m|\vec{r}|\}^2+\{n|\vec{r}|\}^2=6^2+(-3)^2+(2)^2$

$\Rightarrow$ $|\vec{r}|^2(l^2+m^2+n^2)=36+9+4$

$\Rightarrow |\vec{r}|^2=49$ $[\because l^2+m^2+n^2=1]$

$\Rightarrow |\vec{r}|=7$

Putting $|\vec{r}|=7$ in (i), we get

$l=\frac{6}{7}, m=-\frac{3}{7}, n=\frac{2}{7}$

**8.** We know that the perpendicular distance of a point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n}=d$ is given by

$$p=\frac{|\vec{a}\cdot\vec{n}-d|}{|\vec{n}|}$$

Here, $\vec{a}=2\hat{i}-\hat{j}-4\hat{k}, \vec{n}=3\hat{i}-4\hat{j}+12\hat{k}$ and $d=9$.

$\therefore$ the required distance is given by

$$p=\frac{|(2\hat{i}-\hat{j}-4\hat{k})\cdot(3\hat{i}-4\hat{j}+12\hat{k})-9|}{|\sqrt{3^2+(-4)^2+(12)^2}|}$$

$$=\frac{|(6+4-48)-9|}{|\sqrt{169}|}=\frac{47}{13}\text{units}.$$

**9.** The vector $\overrightarrow{AB}\times\overrightarrow{AC}$ is perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\therefore$ Required vector $=\dfrac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|}$

Now, $\overrightarrow{AB}$ = P.V. of $B$ – P.V. of $A$

$=(\hat{i}-\hat{j}-3\hat{k})-(3\hat{i}-\hat{j}+2\hat{k})=-2\hat{i}+0\hat{j}-5\hat{k}$

and $\overrightarrow{AC}$ = P.V. of $C$ – P.V. of $A$

$=(4\hat{i}-3\hat{j}+\hat{k})-(3\hat{i}-\hat{j}+2\hat{k})=\hat{i}-2\hat{j}-\hat{k}$

$$\therefore\quad \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&0&-5\\1&-2&-1\end{vmatrix}$$

$$=(0-10)\hat{i}-(2+5)\hat{j}+(4-0)\hat{k}=-10\hat{i}-7\hat{j}+4\hat{k}$$

$$\Rightarrow\quad |\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-10)^2+(-7)^2+4^2}=\sqrt{165}$$

Hence, required vector $=$

$$\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{|\overrightarrow{AB}\times\overrightarrow{AC}|}=\frac{1}{\sqrt{165}}(-10\hat{i}-7\hat{j}+4\hat{k})$$

**10.** Any plane through the intersection of the two given planes is

$$[\vec{r}\cdot(\hat{i}+3\hat{j}-\hat{k})-5]+\lambda[\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})-3]=0$$

$$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(\lambda-1)\hat{k}]-(5+3\lambda)=0 \quad \ldots(1)$$

If (1) passes through (2, 1, –2), then $\vec{r}=(2\hat{i}+\hat{j}-2\hat{k})$ should satisfy it.

$\therefore (2\hat{i}+\hat{j}-2\hat{k})\cdot[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(\lambda-1)\hat{k}]$

$-(5+3\lambda)=0$

$\Leftrightarrow 2(1+2\lambda)+(3-\lambda)-2(\lambda-1)-(5+3\lambda)=0$

$\Leftrightarrow 2\lambda=2\Leftrightarrow\lambda=1.$

Putting $\lambda=1$ in (1), we get the required equation of the plane as $\vec{r}\cdot(3\hat{i}+2\hat{j})=8$.

**11.** The given equation may be written as

$$\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})=-3$$

$$\Rightarrow\quad \vec{r}\cdot(-6\hat{i}+3\hat{j}+2\hat{k})=3$$

$$\Rightarrow\quad \vec{r}\cdot\vec{n}=3, \text{ where } \vec{n}=(-6\hat{i}+3\hat{j}+2\hat{k})$$

$$\Rightarrow\quad \vec{r}\cdot\frac{\vec{n}}{|\vec{n}|}=\frac{3}{|\vec{n}|}, \text{ where } |\vec{n}|=\sqrt{(-6)^2+3^2+2^2}=7$$

$$\Rightarrow\quad \vec{r}\cdot\frac{(-6\hat{i}+3\hat{j}+2\hat{k})}{7}=\frac{3}{7}$$

$$\Rightarrow\quad \vec{r}\cdot\left(-\frac{6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=\frac{3}{7}$$

Hence, the direction cosines of the normal to the plane are $\left(-\frac{6}{7},\frac{3}{7},\frac{2}{7}\right)$.

**12.** (i) Let $A$, $B$, $C$ be the points with position vectors $\hat{i}+\hat{j}-2\hat{k}, 2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$ respectively.

Then, $\overrightarrow{AB}$ = P.V. of $B$ – P.V. of $A$

$=(2\hat{i}-\hat{j}+\hat{k})-(\hat{i}+\hat{j}-2\hat{k})=\hat{i}-2\hat{j}+3\hat{k}$

and $\overrightarrow{BC}$ = P.V. of $C$ – P.V. of $B$

$=(\hat{i}+2\hat{j}+\hat{k})-(2\hat{i}-\hat{j}+\hat{k})=-\hat{i}+3\hat{j}+0\hat{k}$

A vector normal to the plane containing points $A$, $B$, and $C$ is

$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&3\\-1&3&0\end{vmatrix}=-9\hat{i}-3\hat{j}+\hat{k}$$

The required plane passes through the point having position vector $\vec{a}=\hat{i}+\hat{j}-2\hat{k}$ and is normal to the vector $-9\hat{i}-3\hat{j}+\hat{k}$. So, its vector equation is

$$(\vec{r}-\vec{a})\cdot\vec{n}=0$$

$$\Rightarrow \vec{r}\cdot\vec{n}=\vec{a}\cdot\vec{n}$$

$$\Rightarrow \vec{r}\cdot(-9\hat{i}-3\hat{j}+\hat{k})=(\hat{i}+\hat{j}-2\hat{k})\cdot(-9\hat{i}-3\hat{j}+\hat{k})$$

$$\Rightarrow \vec{r}\cdot(-9\hat{i}-3\hat{j}+\hat{k})=-9-3-2$$

$$\Rightarrow \vec{r}\cdot(9\hat{i}+3\hat{j}-\hat{k})=14$$

(ii) We know that the angle θ between the line $\vec{r}=\vec{a}+\lambda\vec{b}$ and the plane $\vec{r}\cdot\vec{n}=d$ is given by

$$\sin\theta=\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

Here, $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{n}=2\hat{i}-\hat{j}+\hat{k}$

$$\therefore \sin\theta = \frac{(\hat{i}-\hat{j}+\hat{k})\cdot(2\hat{i}-\hat{j}+\hat{k})}{\sqrt{1^2+(-1^2)+1^2}\sqrt{2^2+(-1^2)+1^2}}$$

$$= \frac{2+1+1}{\sqrt{3}\sqrt{6}} = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$$

**13.** Given lines are

$$\vec{r} = 3\hat{i}+8\hat{j}+3\hat{k}+\lambda(3\hat{i}-\hat{j}+\hat{k}) \quad ...(1)$$

and $\vec{r} = -3\hat{i}-7\hat{j}+6\hat{k}+\mu(-3\hat{i}+2\hat{j}+4\hat{k})$ ...(2)

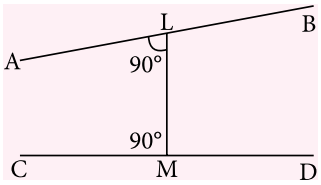
Equations of lines (1) and (2) in cartesian form are

$$AB: \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} = \lambda \quad ...(3)$$

and $CD: \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} = \mu$ ...(4)

Let $L \equiv (3\lambda + 3, -\lambda + 8, \lambda + 3)$

and $M \equiv (-3\mu - 3, 2\mu - 7, 4\mu + 6)$




Direction ratios of $LM$ are

$3\lambda + 3\mu + 6, -\lambda - 2\mu + 15, \lambda - 4\mu - 3$

Since $LM \perp AB$

$\therefore 3(3\lambda + 3\mu + 6) - 1(-\lambda - 2\mu + 15) + 1(\lambda - 4\mu - 3) = 0$

or $11\lambda + 7\mu = 0$ ...(5)

Again $LM \perp CD$

$\therefore -3(3\lambda + 3\mu + 6) + 2(-\lambda - 2\mu + 15) + 4(\lambda - 4\mu - 3) = 0$

or $-7\lambda - 29\mu = 0$ ...(6)

Solving (5) and (6), we get $\lambda = 0, \mu = 0$

$\therefore \quad L \equiv (3, 8, 3), M \equiv (-3, -7, 6)$

Hence, shortest distance $LM$

$$= \sqrt{(3+3)^2+(8+7)^2+(3-6)^2}$$

$= \sqrt{270} = 3\sqrt{30}$ units

Vector equation of $LM$ is

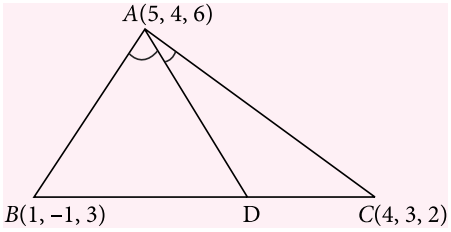
$$\vec{r} = 3\hat{i}+8\hat{j}+3\hat{k}+t(6\hat{i}+15\hat{j}-3\hat{k})$$

**14.** (i) $AB = \sqrt{4^2+5^2+3^2} = 5\sqrt{2}$

$AC = \sqrt{1^2+1^2+4^2} = 3\sqrt{2}$

Since $AD$ is the internal bisector of $\angle BAC$

$$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC} = \frac{5}{3}$$




$\therefore \quad D$ divides $BC$ internally in the ratio 5 : 3

$$\therefore \quad D \equiv \left(\frac{5\times4+3\times1}{5+3}, \frac{5\times3+3(-1)}{5+3}, \frac{5\times2+3\times3}{5+3}\right)$$

or $$D \equiv \left(\frac{23}{8}, \frac{12}{8}, \frac{19}{8}\right)$$

$$\therefore \quad AD = \sqrt{\left(5-\frac{23}{8}\right)^2+\left(4-\frac{12}{8}\right)^2+\left(6-\frac{19}{8}\right)^2}$$

$$= \sqrt{\frac{17^2+20^2+29^2}{8^2}} = \frac{\sqrt{1530}}{8} \text{ units.}$$

(ii) Let $\vec{a} = 4\hat{i}-\hat{j}+8\hat{k}$ and $\vec{b} = -\hat{j}+\hat{k}$.

A unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} \quad ...(1)$$

Now, $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix}$

$$= (-1+8)\hat{i}-(4-0)\hat{j}+(-4-0)\hat{k}$$

$$= 7\hat{i}-4\hat{j}-4\hat{k}$$

$$\therefore |\vec{a}\times\vec{b}| = \sqrt{7^2+(-4)^2+(-4)^2} = \sqrt{81} = 9$$

From (1), a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$

$$= \frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|} = \frac{7\hat{i}-4\hat{j}-4\hat{k}}{9}$$

Hence, the required vector $= \frac{15(7\hat{i}-4\hat{j}-4\hat{k})}{9}$

$$= \frac{5}{3}(7\hat{i}-4\hat{j}-4\hat{k}).$$

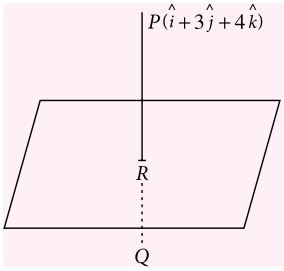
**15.** Let $Q$ be the image of the point $P(\hat{i}+3\hat{j}+4\hat{k})$ in the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$. Then, $PQ$ is normal to the plane. Since $PQ$ passes through $P$ and is normal to the given plane, therefore equation of line $PQ$ is

$$\vec{r} = (\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$$

Since $Q$ lies on line $PQ$, so let the position vector of $Q$ be

$$(\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}-\hat{j}+\hat{k})$$

$$= (1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}$$




Since $R$ is the mid-point of $PQ$. Therefore, position vector of $R$ is

$$\frac{[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(4+\lambda)\hat{k}]+[\hat{i}+3\hat{j}+4\hat{k}]}{2}$$

$$=(\lambda+1)\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}$$

Since $R$ lies on the plane $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})+3=0$

$$\therefore \left\{(\lambda+1)\hat{i}+\left(3-\frac{\lambda}{2}\right)\hat{j}+\left(4+\frac{\lambda}{2}\right)\hat{k}\right\}.(2\hat{i}-\hat{j}+\hat{k})+3=0$$

$$\Rightarrow \quad 2\lambda+2-3+\frac{\lambda}{2}+4+\frac{\lambda}{2}+3=0$$

$$\Rightarrow \quad \lambda=-2.$$

Thus, the position vector of $Q$ is

$$(\hat{i}+3\hat{j}+4\hat{k})-2(2\hat{i}-\hat{j}+\hat{k})=-3\hat{i}+5\hat{j}+2\hat{k}$$

**16.** The given points are $P(6, -4, 4)$, $Q(0, 0, -4)$, $R(-1, -2, -3)$ and $S(1, 2, -5)$. Let us first find the equation of plane through any three of them, say $P$, $Q$, $R$. Equation of any plane through $P(6, -4, 4)$ is

$$A(x-6)+B(y+4)+C(z-4)=0 \quad \ldots(1)$$

Since it passes throught $Q(0, 0, -4)$ and $R(-1, -2, -3)$

$$\therefore \quad -6A+4B-8C=0 \quad \ldots(2)$$

and $-7A+2B-7C=0 \quad \ldots(3)$

Solving (2) and (3) by cross-multiplication, we get

$$\frac{A}{-28+16}=\frac{B}{56-42}=\frac{C}{-12+28} \text{ or } \frac{A}{-12}=\frac{B}{14}=\frac{C}{16}$$

or $\frac{A}{6}=\frac{B}{-7}=\frac{C}{-8}=k \Rightarrow A=6k, B=-7k$ and $C=-8k$

Substituting the values of $A$, $B$ and $C$ in (1), we get

$$6(x-6)-7(y+4)-8(z-4)=0$$

$$\Rightarrow \quad 6x-7y-8z-32=0 \quad \ldots(4)$$

which is the equation of a plane through $P$, $Q$ and $R$.

The four points will be coplanar if the fourth point $S(1, 2, -5)$ lies on (4) *i.e.* if $6(1)-7(2)-8(-5)-32=0$ or if $0=0$ which is true.

So, the four points are coplanar.

Now, two coplanar lines are either parallel to intersecting.

d.r.'s of $PQ$ are $(0-6, 0+4, -4-4)$

*i.e.* $(-6, 4, -8)$ or $(-3, 2, -4)$

d.r.'s of $RS$ are $(1+1, 2+2, -5+3)$

*i.e.* $(2, 4, -2)$ or $(1, 2, -1)$

Since the d.r.'s of $PQ$ and $RS$ are not proportional, So, $PQ$ and $RS$ are not parallel. Hence, $PQ$ intersects $RS$.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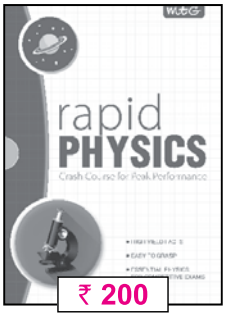

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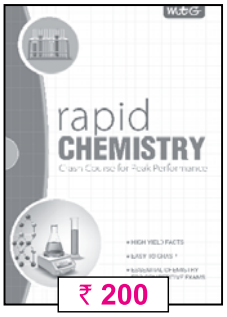

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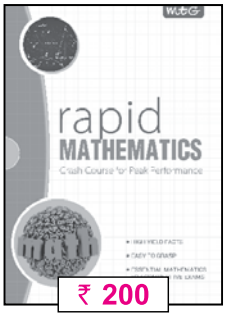

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# WBJEE MOCK TEST PAPER

## UNIT-VI : STATISTICS AND PROBABILITY

The entire syllabus of Mathematics of WB-JEE is being divided into six units, on each unit there will be a Mock Test Paper (MTP) which will be published in the subsequent issues.

### CATEGORY I

**For each correct answer one mark will be awarded, whereas, for each wrong answer, 25% of total marks (1/4) will be deducted. If candidates mark more than one answer, negative marking will be done.**

**1.** The algebraic sum of deviations of 16 observations measured from 25 is 4. The mean of the set is
(a) 4 (b) 404 (c) 25 (d) 25.25

**2.** The marks obtained by 300 candidates in statistics are 46. The mean of the top 100 of them was found to be 70 and the mean of the last 100 was known to be 20. What is the mean of the remaining 100 students?
(a) 48 (b) 46
(c) 70 (d) none of these.

**3.** The sum of the algebraic deviations of a set of values measured from 50 is –10. While from 40 is 70. The number of values is
(a) 8 (b) 7 (c) 6 (d) 5

**4.** The mean weekly salary paid to all employees of a company was Rs. 5000. The mean weekly salaries paid to male and female employees were Rs. 5200 and Rs. 4200 respectively. The percentage of male and female respectively employees in the company is
(a) 60, 40 (b) 80, 20
(c) 20, 80 (d) 70, 30

**5.** The weighted arithmetic mean of first $n$ natural numbers when weights are equal to the corresponding numbers is equal to
(a) $\frac{2n+1}{2}$ (b) $\frac{n(n+1)(2n+1)}{6}$
(c) $\frac{2n+1}{3}$ (d) $\frac{n(2n+1)}{6}$

**6.** The mean deviation about median of the numbers $a, 2a, ..., 50a$ is 50, then $|a| =$
(a) 4 (b) 5 (c) 2 (d) 3

**7.** All the students of a class performed poorly in mathematics. The teacher decided to give grace marks of 10 to each student. Which of the following statistical measures will not change even after the grace marks given?
(a) mean (b) median
(c) mode (d) variance

**8.** The variance of first 50 even natural numbers is
(a) $\frac{833}{4}$ (b) 833 (c) 437 (d) $\frac{437}{4}$

**9.** The standard deviation of 50 values of a variable $x$ is 15. If each value of the variable is divided by –3, then the standard deviation of the new set of 50 values of $x$ will be
(a) 15 (b) –5 (c) 5 (d) –15

**10.** The mean deviation about mean of the set of numbers 7, 9, 24, 14 and 26 is
(a) 7.5 (b) 8 (c) 7.2 (d) 7

**11.** If the sum of the squares of deviations of 25 observations taken from the mean 40 is 900, then the co-efficient of variation is
(a) 20% (b) 12.5% (c) 15% (d) 18%

**12.** If in a distribution, $n = 10$, $\sum x = 20$, $\sum x^2 = 200$, then the value of standard deviation of the distribution is
(a) 2 (b) 16 (c) 6 (d) 4

By : Sankar Ghosh, HOD(Math), Takshyashila. Mob : 09831244397

**13.** Two variables $x$ and $y$ are related by $y = 8 + 2x$; if the s.d. of $x$ is 3, then the s.d. of $y$ will be
(a) 10 (b) 14 (c) 11 (d) 6

**14.** If the mean and co-efficient of mean deviation about mean of a distribution be 40 and 30% respectively, then the mean deviation about mean will be
(a) 12 (b) 15 (c) 20 (d) 24

**15.** The median of a distribution is 60 and mean deviation about median is 24, then the co-efficient of mean deviation of the distribution is
(a) 30% (b) 36% (c) 32% (d) 40%

**16.** 4 boys and 2 girls occupy seats in a row at random. Then probability that the two girls occupy side by side is
(a) 1/2 (b) 1/4 (c) 1/3 (d) 1/6

**17.** Each of $a$ and $b$ can take values 1 or 2 with equal probability. The probability that the equation $ax^2 + bx + 1 = 0$ has real roots, is equal to
(a) 1/2 (b) 1/4 (c) 1/8 (d) 1/16

**18.** A fair six faced die is rolled 12 times. The probability that each face turns up twice is equal to
(a) $\frac{12!}{6!6!6^{12}}$ (b) $\frac{2^{12}}{2^6 6^{12}}$
(c) $\frac{12!}{2^6 6^{12}}$ (d) $\frac{12!}{6^2 6^{12}}$

**19.** An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colour is
(a) $\frac{1}{3}$ (b) $\frac{2}{7}$ (c) $\frac{1}{21}$ (d) $\frac{2}{23}$

**20.** Two decks of playing cards are well shuffled and 26 cards are randomly distributed to a player. Then the probability that the player gets all distinct cards is
(a) $\frac{{}^{52}C_{26}}{{}^{104}C_{26}}$ (b) $\frac{2\times {}^{52}C_{26}}{{}^{104}C_{26}}$
(c) $\frac{2^{13}\times {}^{52}C_{26}}{{}^{104}C_{26}}$ (d) $\frac{2^{26}\times {}^{52}C_{26}}{{}^{104}C_{26}}$

**21.** Two dice are tossed once. The probability of getting an even number at the first die or a total of 8 is
(a) $\frac{1}{36}$ (b) $\frac{3}{36}$ (c) $\frac{11}{36}$ (d) $\frac{5}{9}$

**22.** The probability that at least one of $A$ and $B$ occurs is 0.6. If $A$ and $B$ occur simultaneously with probability 0.3, then $P(A') + P(B')$ is
(a) 0.9 (b) 0.15 (c) 1.1 (d) 1.2.

**23.** An urn contains 8 red and 5 white balls. Three balls are drawn at random. Then the probability that balls of both colours are drawn is
(a) $\frac{40}{143}$ (b) $\frac{70}{143}$ (c) $\frac{3}{13}$ (d) $\frac{10}{13}$

**24.** For two events $A$ and $B$, if $P(A\cup B)=\frac{3}{4}$, $P(A\cap B)=\frac{1}{4}$ and $P(A)=\frac{2}{3}$ then $P(A^c\cap B)$ =
(a) $\frac{5}{12}$ (b) $\frac{3}{8}$ (c) $\frac{1}{12}$ (d) $\frac{3}{12}$

**25.** If $P(A\cap B)=\frac{5}{13}$, then $P(A^c\cup B^c)$ =
(a) $\frac{4}{13}$ (b) $\frac{6}{13}$ (c) $\frac{7}{13}$ (d) $\frac{8}{13}$

**26.** Three numbers are chosen at random without replacement from {1, 2, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is
(a) 1/4 (b) 2/5 (c) 3/8 (d) 1/5

**27.** Let $A$ and $B$ be two events such that $P(A\cup B)^c=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$ and $P(A^c)=\frac{1}{4}$, then the events $A$ and $B$ are
(a) mutually exclusive and independent
(b) equally likely but not independent
(c) independent but not equally likely
(d) independent and equally likely.

**28.** If $C$ and $D$ are two events such that $C\subseteq D$ and $D\neq 0$, then the correct statement among the following is
(a) $P(C|D) < P(C)$ (b) $P(C|D)=\frac{P(D)}{P(C)}$
(c) $P(C|D) = P(C)$ (d) $P(C|D)\geq P(C)$

**29.** A problem in mathematics is given to three students $A$, $B$ and $C$ and their respective probability of solving the problem is 1/2, 1/3, and 1/4. Then probability that the problem is solved is
(a) 3/4 (b) 1/2 (c) 2/3 (d) 7/8

**30.** The probability that a regularly scheduled flight departs on time is 0.9. The probability that it arrives on time is 0.8 and probability that it departs and arrive on time is 0.7. Then the probability that a plane arrives on time, given that it departs on time is
(a) 0.72 (b) 8/9 (c) 7/9 (d) 0.56

## CATEGORY II

**Every correct answer will yield 2 marks. For incorrect response, 25% of full mark (1/2) would be deducted. If candidates mark more than one answer, negative marking will be done.**

**31.** In a bolt factory three machines $A$, $B$ and $C$ manufacture respectively 2000, 2500 and 4000 bolts everyday. Of their outputs 3%, 4% and 2.5% are defective bolts. One bolt is drawn at random from a day's production and it is found to be defective. Then the probability that it was produced by machine $C$ is
(a) $\frac{3}{13}$ (b) $\frac{4}{13}$ (c) $\frac{5}{13}$ (d) $\frac{7}{13}$

**32.** A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is $p$, $0 < p < 1$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, the probability that he did not tick the answer randomly, is
(a) $\frac{3p}{4p+3}$ (b) $\frac{5p}{3p+2}$
(c) $\frac{5p}{4p+1}$ (d) $\frac{4p}{3p+1}$

**33.** There are two coins, one unbiased with probability 1/2 of getting heads and the other one is biased with probability 3/4 of getting heads. A coin is selected at random and tossed. If it shows heads up, then the probability that the unbiased coin was selected is
(a) 2/3 (b) 3/5 (c) 1/2 (d) 2/5

**34.** Let $A$ and $B$ be two events with $P(A^c) = 0.3$, $P(B) = 0.4$ and $P(A \cap B^c) = 0.5$ then $P(B \mid (A \cup B^c)) =$
(a) 1/4 (b) 1/3 (c) 1/2 (d) 2/3

**35.** If $P(A)=\frac{2}{3}$, $P(B)=\frac{1}{2}$ and $P(A \cap B)=\frac{1}{6}$ then find the value of $P(A \cap B') + P(A \cup B)$ is
(a) 3/2 (b) 1/2
(c) 1 (d) none of these

## CATEGORY III

**In this section more than 1 answer can be correct. Candidates will have to mark all the correct answers, for which 2 marks will be awarded. If, candidates mark one correct and one incorrect answer, then no marks will be awarded. But if, candidate makes only correct, without marking any incorrect, formula below will be used to allot marks.**
**2 × (no. of correct response/total no. of correct options)**

**36.** For two events $A$ and $B$, let $P(A) = 0.7$ and $P(B) = 0.6$. The necessarily false statement(s) is/are
(a) $P(A \cap B) = 0.35$ (b) $P(A \cap B) = 0.45$
(c) $P(A \cap B) = 0.65$ (d) $P(A \cap B) = 0.28$

**37.** If $A$ and $B$ are two events and $P(A)=\frac{3}{4}$, $P(B)=\frac{5}{8}$, then
(a) $P(A \cup B) \geq \frac{3}{4}$ (b) $P(A^c \cap B) \leq \frac{1}{4}$
(c) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$ (d) none of these.

**38.** For two events $A$ and $B$, the correct statement is/are
(a) $P(A \cap B) \geq P(A) + P(B) - 1$
(b) $P(A \cap B) \leq P(A)$
(c) $P(A^c \cap B^c) \geq P(A^c) + P(B^c) - 1$
(d) $P(A \cap B) = P(A)\ P(B)$

**39.** $A$ and $B$ are two independent events such that $P(A \cap B)=\frac{1}{12}$ and $P$(either $A$ or $B$) = $\frac{1}{2}$ then
(a) $P(A \cup B)=\frac{1}{2}$
(b) $P(A^c \cap B^c) = \frac{1}{7}$
(c) $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$
(d) $P(A)=\frac{1}{4}, P(B)=\frac{1}{3}$

**40.** If $A$ and $B$ are two events then probability that exactly one of them occurs is
(a) $P(A \cap B^c) + P(A^c \cap B)$
(b) $P(A \cup B) - P(A \cap B)$
(c) $P(A) + P(B) - 2P(A \cap B)$
(d) $P(A^c) + P(B^c) - 2P(A^c \cap B)$

## SOLUTIONS

**1. (d) :** Given that $\sum_{i=1}^{16}(x_i-25)=4$

$\Rightarrow \sum_{i=1}^{16}x_i-400=4 \Rightarrow \frac{1}{16}\sum_{i=1}^{16}x_i=\frac{1}{16}(404)$

$\therefore \bar{x}=25.25$

**2. (a) :** Total marks obtained = 300 × 46 = 13800
Total marks of top 100 = 100 × 70 = 7000
Total marks of last 100 = 100 × 20 = 2000
$\therefore$ Total marks of remaining 100 candidates
= 13800 – 7000 – 2000 = 4800

$\therefore$ Required mean = $\frac{4800}{100}=48$

**3. (a) :** Let the number of values be $n$.

$\therefore \sum_{i=1}^{n}(x_i-50)=-10$ and $\sum_{i=1}^{n}(x_i-40)=70$

$\Rightarrow n(\bar{x}-50)=-10$ ...(i)
and $n(\bar{x}-40)=70$ ...(ii)

Dividing (i) and (ii), $\frac{\bar{x}-50}{\bar{x}-40}=\frac{-10}{70}=-\frac{1}{7}$

$\Rightarrow 7\bar{x}-350=-\bar{x}+40$

$\therefore 8\bar{x}=390 \Rightarrow \bar{x}=48.75$

Also, from (ii), putting $\bar{x}=48.75$

$n=\frac{70}{8.75}=8$

**4. (b) :** Let $n_1$ and $n_2$ be the number of males and females. Then total weekly salary = $(n_1 + n_2) \times 5000$
$\therefore 5000(n_1 + n_2) = 5200n_1 + 4200n_2$
$\Rightarrow 800n_2 = 200n_1$
$\Rightarrow n_1 : n_2 = 800 : 200 = 4 : 1$

$\therefore$ Percentage of males = $\frac{4}{5}\times 100=80$

and percentage of females = $\frac{1}{5}\times 100=20$

**5. (c):** The frequency distribution is

| $x_i$ | $f_i$ | $x_if_i$ |
|---|---|---|
| 1 | 1 | $1^2$ |
| 2 | 2 | $2^2$ |
| 3 | 3 | $3^2$ |
| . | . | . |
| . | . | . |
| $i$ | $i$ | $i^2$ |
| . | . | . |
| . | . | . |
| $n$ | $n$ | $n^2$ |

$\sum_{i=1}^{n}f_i=\frac{n(n+1)}{2}$, $\sum_{i=1}^{n}x_if_i=\frac{n(n+1)(2n+1)}{6}$

$\therefore$ Required mean

$=\frac{n(n+1)(2n+1)}{6}\times\frac{2}{n(n+1)}=\frac{2n+1}{3}$

**6. (a) :** The median of the numbers $a$, $2a$, ..., $50a$ is the mean of 25th and 26th observations.

$\therefore$ Median = $\frac{1}{2}(25a+26a)=\frac{1}{2}\cdot 51a=25.5a$

$\therefore$ Mean deviation about median

$=\frac{1}{50}\sum_{i=1}^{50}|x_i-25.5a|$

$=\frac{1}{50}(|a-25.5a|+|2a-25.5a|+...+|50a-25.5a|)$

$=\frac{2}{50}(24.5a+23.5a+...+0.5a)=\frac{2}{50}(312.5a)$

Given, $\frac{2}{50}(312.5a)=50 \Rightarrow 625a=2500$

$\Rightarrow a=\frac{100}{25}=4$

**7. (d) :** We know that variance is independent of change of origin. Therefore, after giving grace marks to all the students of a class, the variance of their marks will not change.

**8. (b) :** The numbers are 2, 4, 6, 8, ...,100

Variance = $\frac{1}{n}\sum_{i=1}^{50}x_i^2-\bar{x}^2$

Now $\sum_i x_i=2+4+6+...+100$

$=\frac{50}{2}(2+100)=\frac{50\times 102}{2}$

$\therefore \bar{x}=\frac{1}{50}\sum_i x_i=\frac{1}{50}\times\frac{50\times 102}{2}=\frac{102}{2}=51$

Now $\sum_i x_i^2=2^2+4^2+6^2+....+100^2$

$= 2^2(1^2 + 2^2 + 3^2 + .. + 50^2)$
$= 2^2 \times \{50(50 + 1)(2 \times 50 + 1)\}/6$
$= 2^2 \times \{50 \times 51 \times 101\}/6$

$\therefore \frac{1}{n}\sum_i x_i^2=\frac{1}{50}\times\frac{2^2\times 50\times 51\times 101}{6}=\frac{4\times 51\times 101}{6}$

$=\frac{2}{3}\times 51\times 101$

$\therefore$ Variance $= \frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2$

$= \frac{2}{3} \times 51 \times 101 - (51)^2 = 3434 - 2601 = 833$

**9. (c):** Given that $\sigma_x = 15$, $n = 50$

Let $y_i = -\frac{1}{3}x_i$, $i = 1, 2, ..., 50$

$\Rightarrow$ $\text{var}(y) = \frac{1}{9}\text{var}(x) = \frac{1}{9} \times 15^2$ $[\because \sigma_x = 15]$

$\text{var}(y) = 25 \Rightarrow \sigma_y = 5$

$\therefore$ Standard deviation of the new set of 50 values of $x$ will be 5.

**10. (c):** Here $\bar{x} = \frac{7+9+24+14+26}{5} = \frac{80}{5} = 16$

$\therefore$ Mean deviation about mean

$= \frac{1}{5}\sum_{i=1}^{5}|x_i - 16| = \frac{9+7+8+2+10}{5} = \frac{36}{5} = 7.2$

**11. (c):** Given that, $\sum_{i=1}^{25}(x_i - 40)^2 = 900$

$\Rightarrow$ $\frac{1}{25}\sum_{i=1}^{25}(x_i - 40)^2 = \frac{1}{25} \times 900$

$\Rightarrow$ $\sigma_x^2 = 36$ $\therefore$ $\sigma_x = 6$

Now, co-efficient of variation

$= \frac{\sigma_x}{\bar{x}} \times 100 = \frac{6}{40} \times 100 = 15\%$

**12. (d) :** Given that $n = 10$, $\Sigma x = 20$, $\Sigma x^2 = 200$

$\therefore$ $\sigma_x = \sqrt{\frac{1}{n}\Sigma x^2 - \left(\frac{1}{n}\Sigma x\right)^2} = \sqrt{\frac{1}{10} \times 200 - \left(\frac{20}{10}\right)^2}$

$= \sqrt{20 - 4} = \sqrt{16} = 4$

**13. (d) :** The relation between two variables $x$ and $y$ is $y = 8 + 2x$

Let $y_i = 8 + 2x_i$, $(i = 1, 2, .... n)$

$\therefore$ $\bar{y} = 8 + 2\bar{x}$

$\Rightarrow$ $y_i - \bar{y} = 2(x_i - \bar{x})$

$\Rightarrow$ $(y_i - \bar{y})^2 = 4(x_i - \bar{x})^2$

$\Rightarrow$ $\frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 = 4\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$

$\Rightarrow$ $\sigma_y^2 = 4\sigma_x^2 = 4 \times 9 = 36$ $[\because \sigma_x = 3]$

$\therefore$ $\sigma_y = 6$

**14. (a) :** Given that, $\bar{x} = 40$

and co-efficient of mean deviation about mean is 30%.

$\therefore$ $\frac{\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|}{\bar{x}} \times 100 = 30$

$\Rightarrow$ $\frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}| = \frac{30 \times 40}{100} = 12$

Thus, the mean deviation about mean is 12.

**15. (d) :** Given that,

median $= m_e = 60$ and $\frac{1}{n}\sum_{i=1}^{n}|x_i - m_e| = 24$

$\therefore$ The co-efficient of mean deviation is

$\frac{24}{60} \times 100\,\% = 40\,\%$

**16. (c):** Let $A$ be the event that the girls occupy seats side by side.

Now 4 boys and 2 girls can be arranged in a row at random in 6! ways. Therefore sample space contains 6! equally likely event points. Considering 2 girls as one member, we shall have 5 members in all. These 5 members can be arranged in a row in 5! ways and for each such arrangement, 2 girls can arrange among themselves in 2! ways. Therefore, the number of arrangements of 4 boys and 2 girls in which the girls occupy seats side by side is 5! × 2!.

Therefore, by classical definition of probability, we get

$$P(A) = \frac{5! \times 2!}{6!} = \frac{5! \times 2!}{6 \times 5!} = \frac{1}{3}$$

**17. (b) :** The equation $ax^2 + bx + 1 = 0$ has real roots.

$\therefore$ $b^2 - 4a \cdot 1 \geq 0$ $\Rightarrow$ $b^2 \geq 4a$

Now each $a$ and $b$ can take values 1 or 2 with equal probability.

| $a$ | $b$ | $b^2$ | $4a$ |
|---|---|---|---|
| 1 | 1 | 1 | 4 |
| 1 | 2 | 4 | 4 |
| 2 | 1 | 1 | 8 |
| 2 | 2 | 4 | 8 |

The above table shows the possible combination of values of $a$ and $b$. Among the 4 cases, only 1 case, *i.e.* 2nd combination will be favourable for the required event.

Hence, the probability is 1/4.

**18. (c):** Let $A$ be the event that each face turns up twice. Since a six faced die is rolled 12 times therefore, sample space contains $6^{12}$ equally likely event points.

So, the no. of favourable cases is

$${}^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times .... \times {}^{2}C_2$$

$$\therefore \quad P(A) = \frac{{}^{12}C_2 \times {}^{10}C_2 \times {}^{8}C_2 \times ... \times {}^{2}C_2}{6^{12}}$$

$$= \frac{\frac{12!}{2\times 10!} \times \frac{10!}{2\times 8!} \times \frac{8!}{2\times 6!} \times \frac{6!}{2\times 4!} \times \frac{4!}{2\times 2} \times \frac{2}{2}}{6^{12}}$$

$$\Rightarrow \quad P(A) = \frac{12!}{2^6 \times 6^{12}}$$

**19. (b) :** Total number of balls in the urn is 3 + 4 + 2 = 9. Since 3 balls can be selected from 9 balls in ${}^{9}C_3$ ways therefore, sample space contains ${}^{9}C_3$ equally likely event points. Let $A$ denote the event that the 3 balls drawn from the urn at random without replacement are of different colours.

$\therefore$ Favourable cases = ${}^{3}C_1 \times {}^{4}C_1 \times {}^{2}C_1$

$$\therefore \quad P(A) = \frac{{}^{3}C_1 \times {}^{4}C_1 \times {}^{2}C_1}{{}^{9}C_3} = \frac{3\times 4\times 2\times 3!}{9\times 8\times 7} = \frac{2}{7}$$

**20. (d) :** Let $A$ denotes the event that all the 26 cards are distinct.

Since, from 2 decks of playing cards, 26 cards can be drawn randomly in ${}^{104}C_{26}$ ways, therefore the sample space will contain ${}^{104}C_{26}$ equally likely sample points. Now, since, there are 52 distinct cards in a deck and each distinct card is 2 in number.

$\therefore$ 2 decks will also contain 52 distinct cards, two each.

From these, 26 distinct cards are drawn in $2^{26} \times {}^{52}C_{26}$ ways.

$\therefore$ The number of favourable cases = $2^{26} \times {}^{52}C_{26}$

$$\therefore \quad P(A) = \frac{2^{26} \times {}^{52}C_{26}}{{}^{104}C_{26}}$$

**21. (d) :** Let $A$ be the event of getting an even number on the first die and $B$ be the event of getting a total of 8. Total number of sample points in sample space is $6 \times 6 = 36$

Now, number of favourable cases for the event $A$ is $3 \times 6 = 18$ *i.e.*, {(2, 1), (2, 2), (2, 3), ...., (2, 6), (4, 1), (4, 2), (4, 3), ...., (4, 6), (6, 1), (6, 2), ...., (6, 6)}

$$\therefore \quad P(A) = \frac{18}{36}$$

Again, number of favourable cases for the event $B$ is 5 *i.e.*, {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}

$$\therefore \quad P(B) = \frac{5}{36}$$

Now, favourable cases for the event $A \cap B$ are 3 *i.e.*, {(2, 6), (4, 4), (6, 2)}

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$$

**22. (c):** By the problem, we have

$P(A \cup B) = 0.6$ and $P(A \cap B) = 0.3$

Since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow$ $0.6 + 0.3 = P(A) + P(B)$

$\Rightarrow$ $1 - P(A') + 1 - P(B') = 0.9$

$\Rightarrow$ $P(A') + P(B') = 2 - 0.9 = 1.1$

**23. (d) :** Let $A$ be the event that among 3 balls drawn from the urn 2, are red and 1 is white.

Again $B$ be the event that among 3 balls drawn from the urn 1, is red and 2 are white.

Since, 3 balls can be drawn from the urn containing 8 + 5 = 13 balls, in ${}^{13}C_3$ ways. Therefore, there are ${}^{13}C_3$ equally likely sample points in sample space.

$$\therefore \quad P(A) = \frac{{}^{8}C_2 \times {}^{5}C_1}{{}^{13}C_3} \text{ and } P(B) = \frac{{}^{8}C_1 \times {}^{5}C_2}{{}^{13}C_3}$$

$\therefore$ Probability that balls of both colours are drawn is

$P(A \cup B) = P(A) + P(B)$

[$\because$ $A$, $B$ events are mutually exclusive]

$$= \frac{{}^{8}C_2 \times {}^{5}C_1}{{}^{13}C_3} + \frac{{}^{8}C_1 \times {}^{5}C_2}{{}^{13}C_3} = \frac{10}{13}$$

**24. (c):** Given that, for two events $A$ and $B$

$$P(A \cup B) = \frac{3}{4}, \; P(A \cap B) = \frac{1}{4} \text{ and } P(A) = \frac{2}{3}$$

$$\therefore \quad \frac{3}{4} = P(A) + P(B) - \frac{1}{4}$$

$$\Rightarrow \quad P(B) = \frac{3}{4} + \frac{1}{4} - \frac{2}{3} = \frac{4}{12} = \frac{1}{3}$$

Now, $P(A^c \cap B) = P(B) - P(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

**25. (d) :** Here $P(A \cap B) = \frac{5}{13}$

$\therefore$ $P(A^c \cup B^c) = P(A \cap B)^c = 1 - P(A \cap B)$

$$= 1 - \frac{5}{13} = \frac{8}{13}$$

**26. (d) :** Let $A$ denote the event that out of three chosen numbers 6 is maximum and $B$ is the event that 3 is minimum. Since 6 is maximum, therefore we have to select the remaining two numbers from 1, 2, 3, 4, 5

$$\therefore \quad P(A)=\frac{{}^5C_2}{{}^8C_3}=\frac{5}{28}$$

Again, 3 is minimum and 6 is maximum. Therefore, we have to select the remaining numbers from 4, 5

$$\therefore \quad P(A\cap B)=\frac{2}{{}^8C_3}=\frac{1}{28}$$

$$\therefore \quad P(B\,|\,A)=\frac{P(B\cap A)}{P(A)}=\frac{\frac{1}{28}}{\frac{5}{28}}=\frac{1}{5}$$

**27. (c):** Given that $P(A\cup B)^c=\frac{1}{6}$, $P(A\cap B)=\frac{1}{4}$

and $P(A^c)=\frac{1}{4}$

Now, $P(A\cup B)^c=\frac{1}{6}$

$$\Rightarrow \quad P(A^c\cap B^c)=\frac{1}{6} \qquad \text{... (i)}$$

Also, $P(A\cup B)^c=\frac{1}{6} \Rightarrow 1-P(A\cup B)=\frac{1}{6}$

$$\Rightarrow \quad P(A\cup B)=\frac{5}{6}$$

$$\Rightarrow \quad P(A)+P(B)-P(A\cap B)=\frac{5}{6}$$

$$\Rightarrow \quad \frac{3}{4}+P(B)-\frac{1}{4}=\frac{5}{6}$$

$$\Rightarrow \quad P(B)=\frac{5}{6}-\frac{1}{2}=\frac{5-3}{6}=\frac{1}{3}$$

$$\therefore \quad P(B^c)=\frac{2}{3}$$

From (i), $P(A^c\cap B^c)=\frac{1}{6}=\frac{1}{4}\cdot\frac{2}{3}=P(A^c)\cdot P(B^c)$

Thus, the events are independent. Since the probability of $A$ and $B$ are different, so they are not equally likely events.

**28. (d) :** $P(C\,|\,D)=\frac{P(C\cap D)}{P(D)}=\frac{P(C)}{P(D)}$ $[\because C\subseteq D]$

$$\Rightarrow \quad P(C\,|\,D)\geq P(C)$$

$$\left[\because \quad 0<P(D)\leq 1 \quad \therefore \quad \frac{P(C)}{P(D)}\geq P(C)\right]$$

**29. (a) :** Here, $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$ and $P(C)=\frac{1}{4}$

The problem is solved, when any one of them can solve the problem.

$$\begin{aligned}\therefore \quad P(A\cup B\cup C) &= 1-P(A\cup B\cup C)^c\\ &= 1-P(A^c\cap B^c\cap C^c)\\ &= 1-P(A^c)\,P(B^c)\,P(C^c)\\ &= 1-\left\{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\right\}=1-\frac{1}{4}\end{aligned}$$

$$\Rightarrow \quad P(A\cup B\cup C)=\frac{3}{4}$$

Thus, the probability that the problem will be solved is 3/4.

**30. (c):** Let $E_1$ be the event that the plane departs on time and $E_2$ be the event that the plane arrives on time.

$\therefore$ $P(E_1) = 0.9$, $P(E_2) = 0.8$ and $P(E_1\cap E_2) = 0.7$

Now, $P(E_2\,|\,E_1)=\frac{P(E_2\cap E_1)}{P(E_1)}=\frac{0.7}{0.9}=\frac{7}{9}$

**31. (c):** Let us define the events as

$E_1$ : bolt manufactured from machine $A$

$E_2$ : bolt manufactured from machine $B$

$E_3$ : bolt manufactured from machine $C$

Let $A$ be the event that the drawn bolt is defective.

$$P(E_1)=\frac{2000}{2000+2500+4000}=\frac{2000}{8500}=\frac{4}{17}$$

$$P(E_2)=\frac{2500}{2000+2500+4000}=\frac{2500}{8500}=\frac{5}{17}$$

$$P(E_3)=\frac{4000}{2000+2500+4000}=\frac{4000}{8500}=\frac{8}{17}$$

$$P(A\,|\,E_1)=\frac{3}{100},\quad P(A\,|\,E_2)=\frac{4}{100},\quad P(A\,|\,E_3)=\frac{2.5}{100}$$

$$\therefore \quad P(E_3\,|\,A)=\frac{P(E_3)\cdot P(A\,|\,E_3)}{P(E_1)\cdot P(A\,|\,E_1)+P(E_2)\cdot P(A\,|\,E_2)+P(E_3)\cdot P(A\,|\,E_3)}$$

$$=\frac{\frac{8}{17}\times\frac{2\cdot 5}{100}}{\frac{4}{17}\times\frac{3}{100}+\frac{5}{17}\times\frac{4}{100}+\frac{8}{17}\times\frac{2.5}{100}}$$

$$=\frac{8\times 2.5}{12+20+20}=\frac{20}{52}=\frac{5}{13}$$

**32. (c):** Let $E$ denote the event that the student knows the answer.

Therefore $E^c$ denote the event that he does not know the answer.

Let $A$ denote the event that he answer the question correctly.

$\therefore\ P(A|E)=1,\ \ P(A|E^c)=\frac{1}{5}$

Let $P(E) = p$ $\quad\therefore\ P(E^c) = 1 - p$

$\therefore$ Required probability

$$=P(E|A)=\frac{P(A|E)P(E)}{P(A|E)P(E)+P(A|E^c)\times P(E^c)}$$

$$=\frac{1\times p}{1\times p+\frac{1}{5}(1-p)}=\frac{5p}{4p+1}$$

**33. (d) :** Let us define the events as
$E_1$ : the selected coin is unbiased
$E_2$ : the selected coin is biased
$A$ : outcome is head

$\therefore\ P(E_1)=P(E_2)=\frac{1}{2}, P(A|E_1)=\frac{1}{2}$

and $P(A|E_2)=\frac{3}{4}$

$\therefore$ Required probability

$$P(E_1|A)=\frac{P(A|E_1)\cdot P(E_1)}{P(A|E_1)P(E_1)+P(A|E_2)P(E_2)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}\times\frac{1}{2}+\frac{3}{4}\times\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{1}{4}+\frac{3}{8}}=\frac{2}{5}$$

**34. (a) :** Given that
$P(A^c) = 0.3$ $\quad\therefore\ P(A) = 0.7, P(B) = 0.4$
and $P(A \cap B^c) = 0.5$
Now, $P(A \cap B^c) = P(A) - P(A \cap B)$
$\Rightarrow\ 0.5 = 0.7 - P(A \cap B)$
$\Rightarrow\ P(A \cap B) = 0.7 - 0.5 = 0.2$

Now, $P(B|(A\cup B^c))=\frac{P(B\cap(A\cup B^c))}{P(A\cup B^c)}$

$$=\frac{P((B\cap A)\cup(B\cap B^c))}{P(A)+P(B^c)-P(A\cap B^c)}$$

$$=\frac{P((B\cap A)\cup\phi)}{P(A)+1-P(B)-P(A\cap B^c)}$$

$$=\frac{P(B\cap A)}{P(A)+1-P(B)-P(A\cap B^c)}$$

$$=\frac{0.2}{0.7+0.6-0.5}=\frac{0.2}{0.8}=\frac{1}{4}$$

**35. (a) :** Given that

$$P(A)=\frac{2}{3},\ P(B)=\frac{1}{2},\ P(A\cap B)=\frac{1}{6}$$

Now $P(A \cap B') + P(A \cup B)$
$= P(A) - P(A \cap B) + P(A) + P(B) - P(A \cap B)$
$= 2P(A) + P(B) - 2P(A \cap B)$

$$=2\cdot\frac{2}{3}+\frac{1}{2}-2\cdot\frac{1}{6}=\frac{8+3-2}{6}=\frac{3}{2}$$

**36. (c, d) :** Here $P(A) = 0.7$ and $P(B) = 0.6$
Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.7 + 0.6 - P(A \cap B)$
$= 1.3 - P(A \cap B)$
We know that $P(A) \le P(A \cup B) \le 1$
$\Rightarrow\ P(A) \le 1.3 - P(A \cap B) \le 1$
$\Rightarrow\ 0.7 \le 1.3 - P(A \cap B) \le 1$
$\Rightarrow\ 0.3 \le P(A \cap B) \le 0.6$

**37. (a, b, c) :** We are given

$$P(A)=\frac{3}{4},\ \ P(B)=\frac{5}{8}$$

Since $P(A \cup B) \ge P(A)$

$$\Rightarrow\ P(A\cup B)\ge\frac{3}{4}$$

We also have, $P(A \cup B) \le 1$

$$\Rightarrow\ P(A)+P(B)-P(A\cap B)\le 1$$

$$\Rightarrow\ \frac{3}{4}+\frac{5}{8}-1\le P(A\cap B)\ \Rightarrow\ \frac{6+5-8}{8}\le P(A\cap B)$$

$$\Rightarrow\ \frac{3}{8}\le P(A\cap B) \qquad \text{... (i)}$$

Again, $A \cap B \subseteq B$ $\quad\therefore\ P(A \cap B) \le P(B)$

$$\therefore\ P(A\cap B)\le\frac{5}{8} \qquad \text{... (ii)}$$

$\therefore$ From (i) and (ii), we get

$$\frac{3}{8}\le P(A\cap B)\le\frac{5}{8}$$

Now, $P(A^c \cap B) = P(B) - P(A \cap B)$

$$\Rightarrow\ P(A^c\cap B)\le\frac{5}{8}-\frac{3}{8}\quad\left[\because\ \frac{3}{8}\le P(A\cap B)\right]$$

$$\Rightarrow\ P(A^c\cap B)\le\frac{1}{4}$$

**38. (a, b, c) :** We know that $P(A \cup B) \le 1$

$\Rightarrow P(A) + P(B) - P(A \cap B) \le 1$

$\therefore \quad P(A \cap B) \ge P(A) + P(B) - 1$

Again, $A \cap B \subseteq A \quad \therefore P(A \cap B) \le P(A)$

Also, $P(A^c \cap B^c) \le 1$

$\Rightarrow P(A \cup B)^c \le 1 \quad \Rightarrow \quad 1 - P(A \cup B) \le 1$

$\Rightarrow 1 - \{P(A) + P(B) - P(A \cap B)\} \le 1$

$\Rightarrow 1 - P(A) - P(B) + P(A \cap B) \le 1$

$\Rightarrow \{1 - P(A)\} + \{1 - P(B)\} - 1 \le 1 - P(A \cap B)$

$\Rightarrow P(A^c) + P(B^c) - 1 \le P(A \cap B)^c$

$\Rightarrow P(A^c) + P(B^c) - 1 \le 1 \qquad [\because P(A \cap B)^c \le 1]$

Also, $P(A^c) + P(B^c) - 1 \le P(A^c \cap B^c)$

*i.e.* $P(A^c \cap B^c) \ge P(A^c) + P(B^c) - 1$

Now, $P(A \cap B) = P(A) \cdot P(B)$ is only possible when $A$ and $B$ are independent.

**39. (a, c):** Given that $A$ and $B$ are two independent events.

Since, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Rightarrow \quad \frac{1}{2} = P(A) + P(B) - \frac{1}{12}$$

$$\Rightarrow \quad P(A) + P(B) = \frac{7}{12} \qquad \text{... (i)}$$

$$\text{But } P(A \cap B) = \frac{1}{12} \quad \Rightarrow \quad P(A) \cdot P(B) = \frac{1}{12}$$

Now, $\{P(A) - P(B)\}^2 = \{P(A) + P(B)\}^2 - 4P(A)P(B)$

$$= \left(\frac{7}{12}\right)^2 - 4 \times \frac{1}{12} = \frac{49}{12^2} - \frac{4}{12} = \frac{49 - 48}{12^2}$$

$$\therefore \quad \{P(A) - P(B)\}^2 = \left(\frac{1}{12}\right)^2$$

$$\therefore \quad P(A) - P(B) = \frac{1}{12} \qquad \text{... (ii)}$$

(i) + (ii) gives

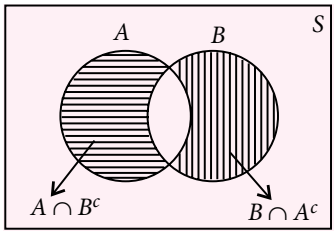
$$2P(A) = \frac{8}{12} = \frac{2}{3} \quad \therefore P(A) = \frac{1}{3}$$

From (i), $P(B) = \frac{7}{12} - \frac{1}{3} = \frac{1}{4}$

$$\therefore \quad P(A) = \frac{1}{3} \text{ and } P(B) = \frac{1}{4}$$

Now, $P(A^c \cap B^c) = P(A^c) \cdot P(B^c) = \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$

**40. (a, b, c) :** Clearly $A \cap B^c$ and $B \cap A^c$ are mutually exclusive.




$\therefore \quad P\{(A \cap B^c) \cup (B \cap A^c)\}$

$= P(A \cap B^c) + P(B \cap A^c)$

From the diagram, it is clear that

$P\{(A \cap B^c) \cup (B \cap A^c)\} = P(A \cup B) - P(A \cap B)$

$= P(A) + P(B) - 2P(A \cap B)$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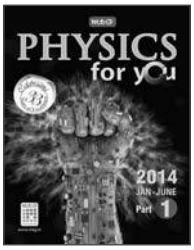


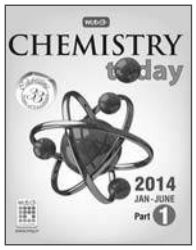


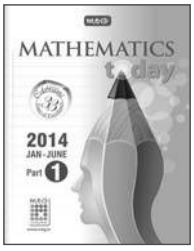


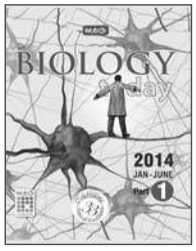


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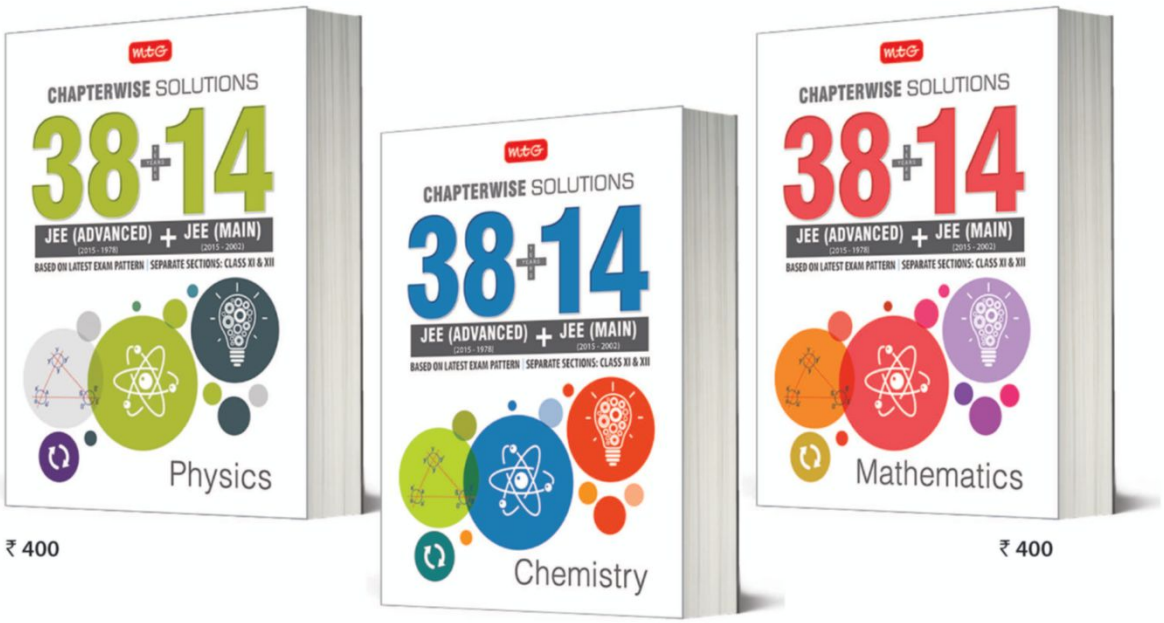


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# MATHS MUSING

## SOLUTION SET-155

**1. (b) :** $\frac{a+b}{2}=\sqrt{ab}+2\rightarrow\sqrt{b}=\sqrt{a}+2$

$(a, b) = (1, 9), (4, 16), (9, 25), (16, 36), (25, 49), (36, 64), (49, 81), (64, 100), (81, 121)$ 9 pairs.

**2. (d) :** Required area

$$= 2\left[\int_0^1(5-x)dx+\int_1^2(5-x)dx\right] -2\left[\int_0^1\left(1-x^2\right)dx+\int_1^2\left(x^2-1\right)dx\right]$$

$= 2(8) - 2(2) = 12$

**3. (d) :**

1 6 11 ..... 96
2 7 12 ..... 97
3 8 13 ..... 98
4 9 14 ..... 99
5 10 15 ..... 100

$a^2 - b^2 = (a + b)(a - b)$

$$\text{Probability} = \left[5\binom{20}{2}+2\binom{20}{1}^2\right]\div\binom{100}{2}=\frac{35}{99}$$

**4. (d) :** $A=\frac{\pi}{7},B=\frac{2\pi}{7},C=\frac{4\pi}{7}$

$\Sigma\tan B\tan C = 1 + \sec A \sec B \sec C$

$$= 1+\frac{1}{\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}}$$

$$= 1+\frac{8\sin\frac{\pi}{7}}{8\sin\frac{\pi}{7}\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}} = 1 - 8 = -7.$$

**5. (a) :** $\overrightarrow{PQ}=\alpha(2\hat{i}+7\hat{j}-5\hat{k})\rightarrow$

$2\alpha = -8 - 3t - 3s$, $7\alpha = -4 + 2t + s$,

$-5\alpha = 8 + 4t - s$

$\therefore\ s = t = \alpha = -1$, $|\overrightarrow{PQ}|^2 = 4 + 49 + 25 = 78$

$|\overrightarrow{PQ}|=\sqrt{78}$

**6. (c, d):** The number of functions

$$=\binom{8-1}{5-1}=\binom{7}{4}=\binom{7}{3}=35=5\times7.$$

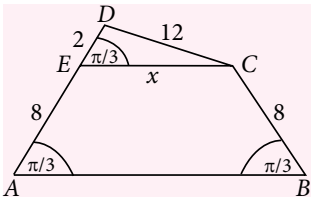
**7. (c, d) :** $\left|re^{i\theta}+\frac{1}{r}e^{-i\theta}\right|^2=4\rightarrow$

$$r^2=2-\cos2\theta\pm\sqrt{3-4\cos2\theta+\cos^2 2\theta}$$

$$\theta=\frac{\pi}{6}\rightarrow r^2=\frac{3\pm\sqrt{5}}{2}\rightarrow r=\frac{\sqrt{5}+1}{2},\frac{\sqrt{5}-1}{2}$$

**8. (a, d) :** $\theta=\frac{\pi}{3}\rightarrow r^2=\frac{5\pm\sqrt{21}}{2},r=\frac{\sqrt{7}\pm\sqrt{3}}{2}$

**9. (6) :**




$EC = x$

In triangle $CDE$, $144=x^2+4-2x\cdot2\cdot\frac{1}{2}$

$$\rightarrow x=1+\sqrt{141}, AB=x+2\cdot8\cdot\frac{1}{2}=9+\sqrt{141}$$

$m = 9$, $n = 141$ , $m + n = 150$, with digit sum 6.

**10. (a) :** $P\rightarrow2, Q\rightarrow2, R\rightarrow5, S\rightarrow4$

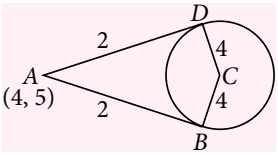
(P) $\lim_{x\to0}1+\frac{f(x)}{x^2}=3\ \therefore\lim_{x\to0}\frac{f(x)}{x^2}=2$

(Q) $2\sin^2 x + \sin x - 1 = 0 \rightarrow \sin x = -1, \frac{1}{2}$

$$\sin x=\frac{1}{2}\rightarrow x=\frac{\pi}{6},\frac{5\pi}{6}$$

$$\sin x=-1\rightarrow x=\frac{3\pi}{2},\ \text{not valid.}$$

(R) Area of $ABCD = 8$




(S) $\frac{x^2}{5}+\frac{(y-3)^2}{9}=1\Rightarrow S = (0, 5), S_1 = (0, 1)$

$P = (3, 3) \Rightarrow \Delta PSS_1 = 6$

**ERRATA**

November Issue Math Musing Problem set 155 page no. 8 Q.No. 2

Q2. The area bounded by the curves $y = |1 - x^2|$ and $y = 5 - |x|$ is

(a) 18 (b) 19 (c) 20 (d) 12

### Solution Sender of Maths Musing

**SET-154**

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**1.** If for $0 < x < \pi/2$, $\exp[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty)\log_e 2]$ satisfies the quadratic equation $x^2 - 9x + 8 = 0$, find the value of $\dfrac{\sin x - \cos x}{\sin x + \cos x}$.

*– Madhav, Assam*

**Ans.** $\because\ 0 < x < \pi/2\ \ \therefore\ 0 < \sin^2 x < 1$

Then $\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty$

$$= \frac{\sin^2 x}{1-\sin^2 x} = \tan^2 x$$

$\therefore$ $\exp[(\sin^2 x + \sin^4 x + \sin^6 x + \ldots \infty)\log_e 2]$

$= \exp\{\tan^2 x \log_e 2\}$

$= \exp\{\log_e 2^{\tan^2 x}\} = e^{\log_e 2^{\tan^2 x}} = 2^{\tan^2 x}$

Let $y = 2^{\tan^2 x}$

$\because$ $y$ satisfies the quadratic equation

$y^2 - 9y + 8 = 0\ \ \therefore\ y = 1, 8$

if $y = 1 = 2^{\tan^2 x}$ then $2^{\tan^2 x} = 2^0$

$\therefore$ $\tan^2 x = 0$

$\therefore$ $x = 0$ (impossible) $(\because x > 0)$

Now if $y = 8 = 2^{\tan^2 x}$ then $2^{\tan^2 x} = 2^3$

$\Rightarrow$ $\tan^2 x = 3\ \ \therefore\ \tan x = \sqrt{3}$

$\therefore$ $\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{\tan x - 1}{\tan x + 1} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$$= \frac{(\sqrt{3}-1)^2}{3-1} = \frac{3+1-2\sqrt{3}}{2} = 2 - \sqrt{3}.$$

**2.** Find the locus of the point from which the two tangents to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ are inclined at an angle $\theta$.

*– Rakesh Sethi, Punjab*

**Ans.** Equation of a tangent to the given ellipse can be written as

$$y - mx = \sqrt{a^2m^2 + b^2} \qquad \ldots(1)$$

If $P(h, k)$ be a point lying on the above tangent, then, we have

$$k - mh = \sqrt{a^2m^2 + b^2}$$

*i.e.,* $(h^2 - a^2)m^2 - 2hkm + k^2 - b^2 = 0 \qquad \ldots(2)$

Let the slope of two tangents or roots of equation (2) be $m_1$ and $m_2$.

Then $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

*i.e.,* $(1 + m_1m_2)^2\tan^2\theta = (m_1 - m_2)^2$

$= (m_1 + m_2)^2 - 4m_1m_2$

*i.e.,* $\left(1 + \dfrac{k^2 - b^2}{h^2 - a^2}\right)^2 \tan^2\theta = \dfrac{4h^2k^2}{(h^2 - a^2)^2} - \dfrac{4(k^2 - b^2)}{(h^2 - a^2)}$

*i.e.,* $(h^2 + k^2 - a^2 - b^2)^2\tan^2\theta = 4(b^2h^2 + a^2k^2 - a^2b^2)$

Putting $(x, y)$ in place of $(h, k)$ gives the equation of the required locus as

$(x^2 + y^2 - a^2 - b^2)^2 \tan^2\theta = 4(b^2x^2 + a^2y^2 - a^2b^2)$.

**3.** Evaluate $\int \ln(\sqrt{1-x} + \sqrt{1+x})\,dx$

*– Swastika, A.P*

**Ans.** We have

$$I = \int \ln(\sqrt{1-x} + \sqrt{1+x})\,dx$$

$$= x\ln(\sqrt{1-x} + \sqrt{1+x}) - \int \frac{x}{\sqrt{1-x} + \sqrt{1+x}}\left(\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}\right)dx$$

$$= x\ln(\sqrt{1-x} + \sqrt{1+x}) - \int \frac{x}{2\sqrt{1-x^2}} \cdot \frac{\sqrt{1-x} - \sqrt{1+x}}{\sqrt{1-x} + \sqrt{1+x}}\,dx$$

$$= x\ln(\sqrt{1-x} + \sqrt{1+x}) - \int \frac{x}{2\sqrt{1-x^2}} \cdot \frac{2 - 2\sqrt{1-x^2}}{-2x}\,dx$$

$$= x\ln(\sqrt{1-x} + \sqrt{1+x}) - \int \frac{\sqrt{1-x^2} - 1}{2\sqrt{1-x^2}}\,dx$$

$$= x\ln(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}}$$

$$= x\ln(\sqrt{1-x} + \sqrt{1+x}) - \frac{1}{2}x + \frac{1}{2}\sin^{-1}x + C$$

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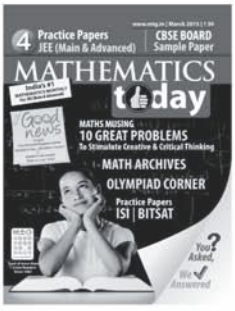

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